The Mathematical Series in Synodic Time Scale. Part III
Time Series of Order 32. Applications and Results.

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Summary

The mathematical series, in Synodic time scale, is given as,

$$y_{(L/2-14)}, y_{(L/2-13)}, \ldots y_{(L/2)}, y_{(L/2+1)}, \ldots y_{(L/2+14)} \qquad \ldots(part1)$$

for one synodic month, where, $y_{(L/2-14)}$ to $y_{(L/2+14)}$ are 29 chained variables in increasing and decreasing order of lunar phase, from nomoon to newmoon, L/2 standing for the fullmoon (FM). The applications indicated modification (*STAT' 2000, August 2000, Poland, abs.*).

The mathematical series, for a Synodic month, in modified form, is given as,

$$y_{(L/2-16)}, y_{(L/2-15)}, \ldots y_{(L/2)}, y_{(L/2+1)}, \ldots y_{(L/2+15)} \qquad \ldots(partI1)$$

Where, $y_{(L/2-16)}$ to $y_{(L/2+15)}$ are 32 chained variables in increasing and decreasing order of lunar phases (*Stability Problem of Stochastic Model, January-February,2001, Hungary, abs.*).

In this paper, the mathematical series is expressed in a simple sequential order of 32 phase groups in Synodic time scale, (*part lll*), from y_{tp1} to y_{tpS} as a Synodic month, given as;

$$y_{tp1}, y_{tp2}, \ldots y_{tp31}, y_{tpS}, y_{tpS+1}, \ldots y_{tp2S}, \ldots y_{tp(n-1)S+31}, y_{tpnS}, \ldots \qquad \ldots(partIll)$$

with seasonality S = 32 (*paper accepted,10th IWMS, August 2-3, 2001, The Netherlands*).

Application to published data (*Inter.Conf.P.V.Epid; 1986, Florida, ext. abs. & Indian Soc. for Agricultural Statistics, 2000, 53, 2, 182-186*) in this scale was proved to be *intrinsic* for noctuids (*5th BS/IMS, May 2000, Mexico*) the *insight* of which was demonstrated earlier (*4th SSC/DST, 1997,WB, India*). The *inferences* on outliers were explained by a *HMM* (*paper accepted for 3rd Bayesian nonparametrics summit, July-August 2001, Michigan*) in this scale, *influencing* generally all species (*paper accepted, ISTAT, CAESAR, June 2001, Rome*) with irregularities probably for not *involving* transformation based on ecological data *(paper accepted, SPA27, July 2001, Cambridge)*. Here this scale is proved to be suitable for *ARIMA models* (*paper accepted for 23rd EMS, August 13-18, 2001, Portugal*) *for all the published Synodic time series data.*

1.The Synodic time series; Part I & Part II

The concept of Synodic time scale is based on the Astronomical Ephemeris, where, in the chapter of the physical observation of the Moon, the portion of the lunar disc illuminated by the Sun i.e. the lunar phases, at 00 hours, is given in the last column, against each calendar date of a year. The discrete series $[y_{ti}]$, at time t_1, t_2, $\ldots t_N$, is given as ;

$$y_{t1}, y_{t2}, \ldots y_{tN} \qquad \ldots(1.1)$$

The time,t, is changed to lunar phase tp, in the discrete series, and is reformulated following the ephemeris, from the beginning of new moon to corresponding no moon, as a Synodic month;

$$y_{tp(.000)}, y_{tp(.002)}, \ldots y_{tp(.039)}, \ldots y_{tp(.098)}, y_{tp(1.00)}, y_{tp(.099)}, \ldots$$
$$\ldots y_{tp(.050)}, \ldots y_{tp(.006)}, y_{tp(.005)}, y_{tp(.004)}, y_{tp(.001)}, \ldots \qquad \ldots a\ Synodic\ month$$

which is, $y_{(L/2-14)}, y_{(L/2-13)} \ldots y_{(L/2)} \ldots y_{(L/2+13)}, y_{2(L/2+14)}, y_{2(L/2-14)} \ldots$

$$y_{(n-1)(L/2)} \ldots y_{(n-1)(L/2+14)}, y_{(n)L/2-14)} \ldots y_{(n)(L/2)} \ldots y_{n(L/2+14)} \qquad \ldots(1.2)$$

where, n is total # of Synodic months, n<N & n29 = N; $y_{(L/2-14)}$ to $y_{(L/2+14)}$ are chained variables in increasing and decreasing order of 29 lunar phases, from no moon to new moon, *L/2* standing for the full moon(FM). The mean of the series is $[\sum y_{sm\,(tpi)}]$ divided by N, for $1 \le i \le 29$ and sm # of Synodic months, N=29×sm. The k^{th} term is, $y_{tpk} = y_{r\,(L/2+v)}$, where, $1 \le r \le sm$ & $0 \le v \le (\pm)14$.

The number of phases in a Synodic month varies from 28 phases to 32 phases, because of missing phases and double phases during full and nomoon period. Also the waxing and waning period of a Synodic month are unequal. All these points are not covered by the series in (1.2) and modification was proved to be necessary in the numerous previous studies done in this scale.

For modification, these $[y_{tpi}]$, are then grouped (*Bowden,1973a, 1973b; Bull. Ent. Res.*), inserting missing phases where required. The reformulated series, for a Synodic month, with 32 phase group is,

$$y_{tp(.000)=newmoon}, y_{tp(.001-.002)}, y_{tp(.003-.006)}, \cdots y_{tp(.098-.099)}, y_{tp(1.00)=fullmoon},$$
$$y_{tp(.099-.098)}, y_{tp(.097-.094)}, \cdots y_{tp(.006-.003)}, y_{tp(.002.001)=nomoon} \quad \ldots(1.3)$$

which is, $y_{(L/2-16)}, y_{(L/2-15)} \cdots y_{(L/2)} \cdots y_{(L/2+15)}, y_{2(L/2-16)}, \cdots y_{2(L/2+15)}, y_{3(L/2-16)}, \cdots$

$$y_{(n-1)(L/2-16)} \cdots y_{(n-1)(L/2+15)}, y_{(n)L/2-16)} \cdots y_{(n)(L/2)} \cdots y_{n(L/2+15)} \quad \ldots(1.4)$$

where, n is total number of Synodic months, n<N & n32=N; $y_{(L/2-16)}$ to $y_{(L/2+15)}$ are chained variables in increasing and decreasing order of 32 lunar phases, *L/2* the fullmoon. The mathematical series (1.4) shows that the two halves of the Synodic month are unequal.This modification makes a Synodic year of 384 phases, gaining 19 data counts than loosing 11 in a year of 365 days with a Synodic month of 29.53 days.The mean of the series is $[\sum y_{SM(tpi)}]$ divided by N, for $1 \le i \le 32$ and # of Synodic months, $N=32\times SM$. The k^th term is, $y_{tpk} = y_{r\,(L/2+v)}$, where, $1\le r \le SM$ & $0\le v\le (\pm)15$, also $v = (-)16$.

2.The Synodic time series; Part III

The mathematical series, in 32 phase groups, can be expressed in a simple sequential order of 32 phase groups. For one Synodic month, the series in (1.3) can be written as ;

$$y_{tp(1)}, y_{tp(2)}, \cdots y_{tp(17)}, y_{tp(18)} \cdots y_{tp(24)}, y_{tp(25)}, \cdots y_{tp(32)} \quad \ldots(2.1)$$

and finally, $y_{tp(1)}, \cdots y_{tp(32)}, y_{tp(32)+1}, y_{tp(32)+2}, \cdots y_{tp(32)+31}, y_{tp2(32)}, y_{tp2(32)+1}, \cdots$

$$y_{tp(n-1)(32)}, y_{tp(n-1)(32)+1}, \cdots y_{tp(n-1)(32)+31}, y_{tpn(32)}, \cdots \quad \ldots(2.2)$$

where, $y_{tp(1)}$ to $y_{tp(32)}$ makes a complete Synodic month, from no moon to new moon, $y_{tp(17)}$ denoting the FM, with seasonality, S = 32. In this way, a lunar phase is less than 24 hours, in general, but there are exceptions when even double phases occur along with phases having more than 24 hours.The reformulated data series, $[y_{ti}]$, thus increases the data counts while time scale is changed from t to tp. Adding extra hidden phases increases the number of data points and the out of sample units are used for double phases, if it occurs while forecasting with a Synodic time series model. The general equation of the Synodic time series is;

$$y_{tp1}, \cdots y_{tpS}, y_{tpS+1}, y_{tpS+2}, \cdots y_{tpS+31}, y_{tp2S}, y_{tp2S+1}, \cdots$$
$$y_{tp(n-1)S}, y_{tp(n-1)S+1}, \cdots y_{tp(n-1)S+31}, y_{tpnS}, \cdots \quad \ldots(2.3)$$

The k^th term is, $y_{tpk} = y_{tp\,(aS+b)}$, where, for the first synodic month, a=0 but $b\ge 1$, $1\le a \le (n-1)$, & $1\le b\le 32$, with a = n and b = 0 for the last term. The mean of the series is $\sum y_{n(tpi)}$ divided by N, for $1 \le i \le 32$ and $N=32\times n$ where n is the number of Synodic months.

3.Applications.

The patterns of the *7 different Time Series*, plotted by *FORTRAN & BASIC* programs (*as in ASMDA, 2001, June 12-15, France*), show irregularity in their structure, except for *TS 1,2,4* & *7*. For *TS 1* the data counts are quite large and for *TS 2,4 & 7*, some other factor/factors may be influencing the pattern to behave less irregularly. All the plotted series, indicate the data to be appropriate, for *ARIMA* models for Synodic Time Series of order 32.