Design of an Optimal Bonus - Malus System in Automobile Insurance

Nicholaos E. Frangos
Athens University of Economics and Business, Department of Statistics
Patission 77
Athens, Greece
nef@aueb.gr

Spyridon D. Vrontos
Athens University of Economics and Business, Department of Statistics
Patission 77
Athens, Greece
svrontos@aueb.gr

1. Introduction

The fundamental principle of insurance consists in forming a pool in which the policyholders put their risks. If the risk structure is heterogeneous the partition of the portfolio of policies into homogeneous classes, with all policyholders belonging to the same class paying the same premium, is necessary. This partition can be done either using a priori classification criteria or a posteriori classification criteria or using both of them. After the use of a priori classification criteria the tariff classes are still heterogeneous, hence the idea of taking under consideration these differences *a posteriori* is introduced. According to the philosophy of a posteriori classification the best predictor of the future number of claims is the driver's past claim behavior. The evolution of the philosophy of a posteriori classification lead to the development of Bonus - Malus Systems (BMS). The BMS penalize insureds responsible for one or more claims by premium surcharges or maluses and reward the policyholders which have a claim free year by awarding them discounts of the premium or bonuses. The main purpose of a BMS besides encouraging policyholders to drive carefully, is to better estimate the individual risks so each policyholder will pay in the long run a premium corresponding to his own claim frequency.

2. Definition of a BMS as a Markov Chain

Loimaranta (1972) gave the following definition of a BMS as a Markov chain. A merit rating system is called BMS if the following assumptions are valid: 1. The policyholders of the portfolio can be partitioned into a finite number of classes so that the annual premium depends only on the class. 2. The policyholder's class is uniqely defined by the class of the previous period and the number of claims occurred during the previous period. In order to specify a BMS the knowledge of the following three factors is necessary: 1.Transition rules, which are the rules that give the new class when the old class and the number of claims for the period are known. 2.The premium scale. 3.The initial class C_{i_0}, in which the new policyholders are placed. As we will always assume that the claim frequency is stationary in time the chain is homogeneous. The BMS defined in the above way forms a Markov process in which the states of the chain are the different BMS classes.

2..1 Efficiency of a BMS

A variety of BMS is in use and the development of methods to evaluate and compare different BMS is necessary. The first method to achieve this was the asymptotic efficiency or the elasticity of the mean stationary premium b with respect to claim frequency $\eta(\lambda)$ of Loimaranta (1972) which was defined as following:

$$\eta(\lambda) = \frac{\frac{db}{b}}{\frac{d\lambda}{\lambda}} = \frac{d\log b}{d\log\lambda}.$$

In an ideal situation the BMS is perfectly efficient and a fractional change in the claim frequency $\frac{d\lambda}{\lambda}$ will cause an equal fractional change in the premium $\frac{db}{b}$, but in most of the cases the change in the premium is smaller from the change in the claim frequency.

Lemaire (1995) defines the transient efficiency as following: Consider a discount factor $\beta < 1$ and denote $\nu_i^{(n)}(\lambda)$ the discounted expectation of all payments made by a policyholder with claim frequency λ, placed at the class C_i at the beginning of his n-year driving lifetime. The transient efficiency, denoted as $\mu_i(\lambda)$, of a BMS is then defined as:

$$\mu_i(\lambda) = \frac{\frac{d\nu_i(\lambda)}{\nu_i(\lambda)}}{\frac{d\lambda}{\lambda}} = \frac{d\ln\nu_i(\lambda)}{d\ln\lambda},$$

where $\mu_i(\lambda)$ is the efficiency of the discounted expectation of the payments with respect to claim frequency. One of the advantages of transient efficiency over the asymptotic efficiency is that it could be used to select the starting class as this one that maximizes $\mu_i(\lambda)$ for an average value of λ.

3. Optimal BMS with a Frequency Component

The portfolio is considered to be heterogeneous and all policyholders have constant but unequal underlying risks to have an accident. We consider for the number of claims k given the parameter λ that $k|\lambda \sim Poisson(\lambda)$ where λ is characterizing the accident proneness of each policyholder. If $\lambda \sim Gamma(\alpha, \beta)$ then it is known for the unconditional distribution of the number of claims that $k \sim Negative\ Binomial(\alpha, \beta)$ and the posterior structure function for a policyholder with k accidents in t years will be $u(\lambda \mid k) \sim Gamma\ (\alpha+k, \frac{\beta}{\beta t+1})$. Lemaire (1995) develop the design of an optimal BMS as follows: Consider a policyholder observed for t years and denote by k_i the number of his claims during year i. Using the information from each group of observations $(k_1, k_2, ..., k_t)$ we must find $\lambda_{t+1}(k_1, k_2, ..., k_t)$, which will be the best estimate of the claim frequency λ of the policyholder at time $t+1$. Denoting as $R_{t+1} = R_{t+1}(\lambda, \lambda_{t+1})$ the risk function of the actuary at time $t+1$ equal to the expectation of the loss $L_{t+1}(\lambda, \lambda_{t+1})$ that he incurs when he takes decision $\lambda_{t+1}(k_1, k_2, ..., k_t)$ while the true value is λ it is known that using the quadratic error loss function the choice of λ which minimizes the loss incurred is the mean of the posterior structure function. Thus using the net premium principle the policyholder will have to pay a premium equal to

$$P = \lambda_{t+1}(k_1, ..., k_t) = E(\lambda | k_1, ..., k_t) = (\alpha + k) \cdot (\frac{\beta}{\beta t + 1}) \tag{1}$$

The optimal BMS has some very important properties: 1. Every policyholder has to pay a premium exactly proportional to the estimate of his claim frequency. 2. The system is financially balanced. Every single year the bonuses given are equal to the maluses and the economic security of the insurer is guaranteed. 3.The premium can be written as a special case of the well known simple B$\ddot{u}$hlmann credibility model, B$\ddot{u}$hlmann (1967), which denotes that the posterior mean is the weighted average of the a priori premium and the observations. This property give us the opportunity to evaluate the asymptotic and the transient efficiency using an approximation presented by Sundt (1989). Sundt proves that an optimal BMS that can be written in the form of the simple B$\ddot{u}$hlmann credibility model has an asymptotic efficiency equal to 1, which means that the BMS is perfectly efficient. This result proves the superiority of an optimal BMS in comparison with a Markovian BMS.

4. Optimal BMS with a Frequency and a Severity Component

The optimal BMS with a frequency component penalizes the number of claims without taking into acount the size of loss that the claim incurrs. In this way the insureds who had an accident with a small size of loss are enforced to pay the same premium with the insureds who had an accident with a big loss. In order to develop the design of an optimal BMS using both the frequency and the severity component we assume that the number of claims is independent from the severity. For the frequency component we will use the same structure as in the previous section. Let's consider now the severity component. Let x be the size of the claim of each insured. We consider as y the mean claim size for each insured and we assume for the conditional distribution of the size of each claim given the mean claim size $x|y$ that $x|y \sim Exponential(y)$. If we have for the mean claim size y that $y \sim Inverse\ Gamma(s, m)$ then it can be proved for the unconditional distribution of the claim size x in the portfolio that

$x \sim Pareto(s, m)$. When the $Pareto(s, m)$ distribution has a good fit to the claim size of a portfolio of policyholders we can apply the above process.

Consider that the policyholder is in the portfolio for t years and that the number of claims he has done in the year i is denoted with k_i and by x_i is denoted the aggregate claim amount for the year i. Then the information we have for his claim size history will be in the form of a vector $x_1, x_2, ..., x_t$ and the aggregate claim amount, or the total claim amount for the specific policyholder over the t years that he is in the portfolio will be equal to $\sum_{i=1}^{t} x_i$. The posterior distribution of the mean claim size y given the sizes of the claims will be $y|x_1, ..., x_t \sim Inverse\ Gamma(s + t, m + \sum_{i=1}^{t} x_i)$. Thus the net premium that must be paid from a policyholder or a group of policyholders who in t years of observation have produced k claims with aggregate claim amount equal to $\sum_{i=1}^{t} x_i$ will be equal to the product of $\lambda_{t+1}(k_1, ..., k_t)$ and the mean of the posterior distribution of the mean claim size y given the sizes of the claims $y_{t+1}(x_{1,...,}x_t)$, so it will be equal to

$$P = (\alpha + k) \cdot (\frac{\beta}{\beta t + 1}) \cdot \frac{m + \sum_{i=1}^{t} x_i}{s + t - 1} \tag{2}$$

The optimal BMS with a frequency and a severity component has the following properties: 1. The system is fair as each insured pays a premium proportional not only to his claim frequency but also to the exact size of his claims. 2. The system is financially balanced. 3.The frequency and the severity are computed each one as special cases of the well known simple Bühlmann credibility model. For more on such a system see Vrontos (1998).

5. Application

The models discussed are applied in a data set that the Statistical Service of the Association of Insurance Companies of Greece provided us. This data set concern all the passenger cars insured the year 1994 for third party liability in all the Greek insurance companies.

5..1 Criticism of the Current Greek BMS - A Proposal

The current Greek BMS is constituted from 16 classes. The new policyholders who enter the system are placed in class 10. The policyholders who had a claim free year will be placed in the class below from their previous class and they will be awarded of a 10 percent discount or bonus. The policyholders who had one or more accidents, will be surcharged of a 20 percent malus for each accident they had and they will have an increase of two classes for each accident they had. We present the current Greek BMS in Table 1.
Table 1: The Current Greek Bonus - Malus System

Class	Premium	Class	Premium	Class	Premium
5	50	11	110	17	170
6	60	12	120	18	180
7	70	13	130	19	190
8	80	14	140	20	200
9	90	15	150		
10	100	16	160		

The Greek BMS is designed for an average claim frequency equal to 33 percent while the average claim frequency in Greece is a little less than 10 percent. This has lead to the imbalance of the BMS as the majority of the insureds is placed in the Bonus classes and the discounts that are given are much more than the maluses. The imbalance of the Greek BMS is possibly the most important reason for the lack of economic security that characterizes it and the most important reason for its change. The Greek BMS penalizes the number of accidents disregarding their severity that is the size of the loss the accidents incur. In this way an accident with a small loss is penalized equally with an accident with a big loss.

The asymptotic efficiency of the mean claim frequency for 1994 in Greece is equal to $\eta(0.0823) = 0.08714$. This means that for two policyholders, the first with claim frequency $\lambda = 0.10$ and the second with claim frequency $\lambda = 0.12$ instead of having the second driver paying over a long period a premium 20 percent higher than the premium of the first driver (as it would happen in a perfectly efficient BMS), the change in the premium will be smaller from the change in the claim frequency and specifically equal to 0.08714 times the change in the claim frequency. The average claim frequency worldwide is considered to be equal to $\lambda = 0.10$ and the asymptotic efficiency for this value is equal to $\eta(0.10) = 0.12077$. Respectively for the transient efficiency we have that $\mu_{10}(0.08228) = 0.05771$ and $\mu_{10}(0.10) = 0.07834$.

We will present now an alternative BMS defined as a Markov chain for the Greek automobile insurance with the same number of classes and the same premiums with the current Greek BMS but with more severe transition rules and more efficient than that. For more on this see Vrontos (1998). The transition rules are: 1. A driver goes down one class only after two claim free years. If a driver with claim free year has an accident in the second year then he is not losing the advantage of the previous claim free year. 2. A driver with an accident that incurs a loss smaller than 40000 dr is staying in the same class and a driver with an accident that incurs a loss bigger than 40000 dr goes up two classes. 3. A driver with more than one accidents goes up two classes for each accident he has. We calculated the asymptotic efficiency for this BMS and we find that it is more efficient than the current Greek BMS. That is $\eta(0.10) = 0.306$ while for the current Greek BMS $\eta(0.10) = 0.12077$ and $\eta(0.0823) = 0.2083$, while for the current Greek BMS $\eta(0.0823) = 0.0874$.

5..2 Optimal BMS with a Frequency Component

The number of the claims of each policyholder is distributed according the Negative Binomial with parameters $\hat{\alpha}$=0.34854 and $\hat{\beta}$=0.23607. The BMS will be defined from (1) and is presented in Table 2. This optimal BMS is generous with the good drivers and strict with the bad drivers as the bonuses given for one claim free year are 19 percent of the basic premium and the driver who has one accident will have to pay a malus of 213 percent of the basic premium.

Table 2. Optimal Bonus Malus System - Net Premium Principle

Year	Number of claims					
t	0	1	2	3	4	5
0	100					
1	81	313	545	777	1009	1241
2	68	263	458	653	848	1042
3	59	227	394	562	730	898
4	51	199	347	494	642	789
5	46	177	309	441	572	704

The asymptotic and the transient efficiency of the optimal BMS obtained for the most common values of the age of the policy using the approach of Sundt (1988). According the transient efficiency the optimal BMS is significantly more efficient than the current Greek BMS, for the first 30 years of the age of the policy the transient efficiency of the optimal BMS ranges from 0.63 to 0.21 while for the current Greek BMS it is equal to 0.057. The asymptotic efficiency for all the optimal BMS that can be written in the simple B$\ddot{u}$hlmann credibility model is equal to 1 while the asymptotic efficiency for the mean claim frequency of the current Greek BMS is equal to $\eta(0.0823) = 0.08714$.

5..3 Design of an optimal BMS Using Both Frequency and Severity Component

The number of the claims of each policyholder is distributed according the Negative Binomial with parameters $\hat{\alpha}$=0.34854 and $\hat{\beta}$=0.23607. The claim size x of each policyholder who had an accident is distributed according the Pareto $s = 2.843$ and $m = 518079$. The BMS is determined using (2). For more on this see Vrontos (1998). We will illustrate here how this optimal BMS permits the discrimination of the premium with respect to the severity of the claim. In Table 3 we see the premiums that must be paid for various number of claims and claim sizes when the policyholder is observed for one year.

Table 3. Comparison of Premiums for Various Number of Claims and Claim Sizes

	Number of claims				
Claim Size	1	2	3	4	5
250000	69579	121176	172772	224368	275964
500000	92227	160617	229007	297397	365787
1000000	137521	239499	341477	443455	545433
2000000	228110	397264	566417	735570	904724

A policyholder who had one claim with claim size 250000 will have to pay a premium of 69579 dr and a policyholder who had one claim with claim size 1000000 will have to pay a premium of 137521 dr.

REFERENCES

Buhlmann, H. (1967). Experience rating and credibility, *ASTIN Bulletin* 4, 199-207.

Lemaire, J. (1995). Bonus - Malus Systems in Automobile Insurance, Kluwer Academic Publishers, Massachusetts.

Loimaranta, K. (1972). Some Asymptotic Properties of Bonus systems, *ASTIN Bulletin* 6, 233-245.

Sundt, B. (1988). Credibility estimators with geometric weights, *Insurance: Mathematics and Economics* 7, 113-122.

Vrontos, S. (1997). Design of Optimal Bonus- Malus Systems in Automobile Insurance, M.Sc. Dissertation, Department of Statistics, Athens University of Economics and Business.

RESUME

In automobile third-party liability insurance the policyholders have not all the same risk to have an accident. The premium that each policyholder is paying has to be analogous to his underlying risk to have an accident. Bonus - Malus Systems (BMS) address this problem in a satisfactory way. We consider the design of an optimal BMS based on the claim frequency of each policyholder, and the design of an optimal BMS based on the claim frequency and the claim severity for each policyholder. We evaluate the current Greek BMS and we propose an optimal and a Markovian BMS which are more efficient in comparison with the current Greek BMS.