M Quasi-likelihood Estimation for Nonlinear Time Series Models

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1. Introduction

The quasi-likelihood estimation has been very useful tool for estimating the parameters of interest when the underlying distribution is not fully specified except for the mean and the variance structures. The original idea of quasi-likelihood estimation was proposed by Wedderburn(1974) and this has been widely used in generalized linear models. Godambe(1985) showed that Wedderburn(1974)'s quasi-likelihood function satisfied the optimal properties of estimating functions for discrete time stochastic processes with martingale structure.

Huber(1964) proposed robust estimation under the heavy-tailed distributions of the errors for regression models. Denby and Martin(1979) applied Huber's M estimator to AR(Autoregressive) models and compared the efficiency with least squares estimator. Also, Martin and Yohai(1985) summarized the concepts and methods of robustness in linear time series model. But not much research on robust estimation is in process for nonlinear time series models.

In this paper, we introduce the M quasi-likelihood estimating function to estimate parameters in a class of nonlinear time series. Asymptotic properties of the estimator which is the solution of the M quasi-likelihood estimating function will be presented.

2. M Quasi-likelihood estimating functions for Nonlinear Time series Models

Various authors have considered estimators which are solutions of the estimating equations and investigated their asymptotic properties under the assumption that the model is correct. Consider the following general form of nonlinear time series model satisfying a recursive equation.

$$X_t = m_t(X_{t-1}; q) + e_t \cdot n_t^{1/2}(q) \quad \text{t = 1,..,n} \qquad (2.1)$$

where $\{e_t\}$ is an unobserved sequence of i.i.d. random variables with mean zero and variance s^2.

Also $m_t(q) = E(X_t | F_{t-1})$, $n_t(q) = Var(X_t | F_{t-1})$, $\underline{X}_{t-1} = (X_{t-1}, \ldots, X_{t-p})$, $p > 1$, q is an unknown $p \times 1$ vector parameter. It is assumed that $\{X_t\}$ is stationary and ergodic. Now we define the M quasi-likelihood estimating function as following.

$$S_n(q) = \sum_{t=1}^{n} g(X_t; q) = \sum_{t=1}^{n} y_1\left(\frac{X_t - m_t(\underline{X}_{t-1}, q)}{v_t(q)}\right) y_2\left(\frac{d\, m_t(\underline{X}_{t-1}; q)}{d\, q}\right) \qquad (2.2)$$

$$= \sum_{t=1}^{n} \begin{pmatrix} g_i(X_t; q) \\ \mathrm{M} \\ g_p(X_t; q) \end{pmatrix},$$

where $m_t(\underline{X}_{t-1}, q) = \mathrm{E}(X_t \mid F_{t-1})$, $n_t(q) = Var(X_t | F_{t-1})$ and F_{t-1} is a s -field generated by $X_{t-1}, \ldots, X_{t-p}$, $t \geq 1$. y_1 and y_2 are bounded functions and differentiable in q .

An estimator $\hat{q}_n$ obtained by solving the estimating equation $S_n(q) = 0$ is called as the M quasi-likelihood estimator.

<Theorem 2.1> Let $\hat{q}_n$ be a consistent solution of $S_n(q) = 0$, Then under the regularity conditions, we have

$$\sqrt{n}(\hat{q}_n - q) \xrightarrow{d} N(0, v^{-1}(q)), \qquad (2.3)$$

where the variance $v(q)$ is given by $[A^T(q) B(q)^{-1} A(q)]$ with $A = ((a_{ij}))$ and $B = ((b_{ij}))$, and $a_{ij} = \mathrm{E}_q(\partial g_i / \partial q_j)$ and $b_{ij} = \mathrm{E}_q(g_i, g_j)$,respectively.

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