How to Relate Robustness Measures with Reliability

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1. Introduction

- Experimental design has become one of the primary tools for achieving quality of manufactured products.
- Because of a division in the disciplines of quality and reliability, this important tool has not been used to extensively to achieve reliability
- In this presentation, we describe how experimental designs can be used to achieve reliability

2. Experiments with failure time data

<u>Example 1</u> Rolling Ball Bearing Lifetimes from Hellstrand(1989)
<u>Example 2</u> Light Experiments from Taguchi(1987)
<u>Example 3</u> Thermostast Experiment from Bullington et al(1993)
<u>Example 4</u> Drill Bit Experiment

3. Regression Model for failure time data

3.1. Distribution

(1) failure time t takes the nonnegative values

$\Rightarrow$ a standard linear regression modeling of t is not appropriate

- lognormal density function $t \sim LN(m, s^2)$

  Weibull density function

(2) the most obvious choice is the log transformation $Y = \ln(t)$

- $t \sim LN(m, s^2) \Rightarrow y = \ln t \sim N(m, s^2)$
- $t \sim Weibull(\ l, j\ )$

  $\Rightarrow y = \ln t \sim extrme\,value\,distribution(m = -\ln\ l,\ s = j^{-1})$

3.2. Model

$$Y_i = \ln(t_i) = \underline{x_i^T\, b} + se_i$$

$$= b_0 + b_1 x_{i1} + \Lambda\ + b_p x_{ip} + se_i\ ,\quad i = 1, 2, \Lambda\ , n$$

- the $\{t_i\}$ are the failure times
- $\underline{x_i^T} = (1, x_{i1}, x_{i2}, \Lambda\ , x_{ip})$ are the vectors of covariate values
- $\underline{b} = (b_0, b_1, \Lambda\ , b_p)$ are the vectors of regression parameters
- s is the scale parameter
- the errors $\{s_i\}$ are i.i.d.

(Note) $\{s_i\}$ have standard extreme-value distribution if t_i follows a Weibull distribution

$\{s_i\}$ have independent $N(0,1)$ distribution if t_i follows a lognormal distribution

4. Likelihood Approach

(1) MLE

- The likelihood function
  - The lognormal regression model

$$L(b, s^2) = \prod_{i \in FAIL} (2ps^2)^{-1/2} \exp\left\{\left(-\frac{1}{2}\right)\left[\left(y_i - x_i^T b\right)/ s\right]^2\right\} \times \prod_{i \in CEN} \left\{1 - \Phi\left[\left(y_i - x_i^T b\right)/ s\right]\right\}$$ the

Weibull regression model

$$L(b, s^2) = \prod_{i \in FAIL} (1/s)\exp\left\{\left[\left(y_i - x_i^T b\right)/ s\right] - \exp\left[\left(y_i - x_i^T b\right)/ s\right]\right\} \times \prod_{i \in CEN} \exp\left\{-\exp\left[\left(y_i - x_i^T b\right)/ s\right]\right\}$$

MLE for $(\underline{\hat{B}}, \hat{s}^2)$ can be obtained by maximizing $L(b, s^2)$ with respect to $(\underline{b}, s^2)$

(2) Likelihood ratio tests

- Likelihood ratio tests provide a method for assessing significance of the *i*th parameter in q

$\Gamma = -2 \ln \dfrac{L(\hat{q}_{(-i)})}{L(\hat{q})}$, where the *i*th covariate is dropped from the model and $q_{(-i)}$ are the corresponding parameters.

- Under $H_0 : q_i = 0$, Γ is distributed asymptotically as c^2 (df=1).

5. Anaysis of reliability experimental data

5.1. 2^k factorial design

(1) Weibull Regression Model

$$y_i = \ln(t_i) = b_0 + b_1 x_{i1} + b_2 x_{i2} + b_3 x_{i3} + se_i, \quad i = 1,2,\Lambda\ ,8$$

MLEs, Likelihood Ratio(LR), and P-values

5.2. 2^{5-2} fractional factorial design(Taguchi)

Light Experiments

(1) Lognormal Regression Model

$$y = \ln(t) = b_0 + b_1 A + b_2 B + b_3 C + b_4 D + b_5 E + b_{12}(AB) + se,$$

(2) MLEs, Likelihood Ratio(LR), and P-values

5.3. 12-run Plackett-Burman design with ten replicates

Thermostast Experiment

(1) Lognormal Regression Model 1

$$y = \ln(t) = b_0 + b_1 A + b_2 B + b_3 C + b_4 D + b_5 E + b_6 F + b_7 G + + b_8 H + b_9 I + b_{10} J + b_{11} K + se$$

(2) MLEs, Likelihood Ratio(LR), and P-values

(3) Further Analysis

(4) Lognormal Regression Model 2

$$y = \ln(t) = b_0 + b_1 A + b_2 (EH) + b_3 C + b_4 D + b_5 E + b_6 F + b_7 G + + b_8 H + b_9 I + b_{10} J + b_{11} K + se$$

(5) MLEs, Likelihood Ratio(LR), and P-values

5.4. Taguchi Design

Cross Array and faiure Time Data(with censoring time of 3000)

(1) Design :

(2) Factors

(3) Use Weibull Regression Model

(4) MLEs, Likelihood Ratio(LR), and P-values

References

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