Bayesian Inference for the Reliability of a Multicomponent Stress-Strength System Using Noninformative Priors

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1. Introduction

Suppose a system, made up of k identical components, functions if r or more of the components simultanuously operate. We assume that the component strengths, $Y_1, \cdots, Y_k$, are independently and identically distributed(i.i.d.) as a common generalized gamma distribution, $GG(\eta_1, \beta, p)$, with density function $f(x|\eta_1, \beta, p) = \frac{\beta}{\Gamma(p)}\eta_1^{-p\beta}x^{p\beta-1}e^{-(\frac{x}{\eta_1})^\beta}$, $x > 0, \eta_1, \beta, p > 0$. We further suppose that the system is subject to a stress, X, which is an independent random variable with another generalized gamma distribution, $GG(\eta_2, \beta, p)$. Then, by some manuplation, the system reliability, which is the probability that the system operates satisfactorily, is given by

$$\begin{aligned} R &= P(\text{at least } r \text{ of the } Y_1, \cdots, Y_k \text{ exceed } X) \\ &= \sum_{i=r}^{k} \binom{k}{i} \int_0^\infty [1 - I(p, u)]^i [I(p, u)]^{k-i} \frac{1}{\Gamma(p)} \theta_1^p u^{p-1} e^{-\theta_1 u} du, \end{aligned} \tag{1.1}$$

where $\theta_1 = (\frac{\eta_2}{\eta_1})^\beta$ and $I(p, u) = \int_0^u \frac{1}{\Gamma(p)} v^{p-1} e^{-v} dv$. Note that R depends only on θ_1 and p. In this paper, we consider Bayesian inference for R in the case when p is known.

2. Noninformative Priors and Posterior Analysis

Let $X_1, X_2, \cdots, X_m$ be i.i.d. as $GG(\eta_1, \beta, p)$ and independently, $Y_1, Y_2, \cdots, Y_n$ be i.i.d. as $GG(\eta_2, \beta, p)$, where p is known. Consider the reparametrization $\theta_1 = (\frac{\eta_2}{\eta_1})^\beta$, $\theta_2 = \eta_1^{\frac{m}{m+n}} \eta_2^{\frac{n}{m+n}} e^{\frac{\gamma_1}{p\Gamma(p)} \frac{1}{\beta}}$,

$\theta_3 = \beta[mnp(log(\frac{\eta_2}{\eta_1})^\beta)^2 + (m+n)^2\gamma_*]^{-\frac{1}{2}}$, *where* $\gamma_* = 1 + \frac{\gamma_2}{\Gamma(p)} - \frac{\gamma_1^2}{p\Gamma^2(p)}$ *with* $\gamma_i = \int_0^\infty (logz)^i z^p e^{-z} dz$, $i = 1, 2$. *Then the Fisher information matrix for* $(\theta_1, \theta_2, \theta_3)$ *is given by*

$$I = Diag\{I_{11}, I_{22}, I_{33}\},$$

where $I_{11} = mnp(m+n)\gamma_*\theta_1^{-2}[mnp(log\theta_1)^2 + (m+n)^2\gamma_*]^{-1}$, $I_{22} = (m+n)p\theta_2^{-2}\theta_3^2[mnp(log\theta_1)^2 + (m+n)^2\gamma_*]$, *and* $I_{33} = \frac{1}{m+n}\theta_3^{-2}[mnp(log\theta_1)^2 + (m+n)^2\gamma_*]$.

Theorem 2.1. *The Jeffreys' prior for* $(\theta_1, \theta_2, \theta_3)$ *is*

$$\pi_J(\theta_1, \theta_2, \theta_3) \propto \theta_1^{-1}\theta_2^{-1}[mnp(log\theta_1)^2 + (m+n)^2\gamma_*]^{\frac{1}{2}}. \quad (2.1)$$

Following Datta and Ghosh(1995), we can obtain a reference prior for $(\theta_1, \theta_2, \theta_3)$ *when* θ_1 *is the parameter of interest and* (θ_2, θ_3) *are nuisance parameters.*

Theorem 2.2. *A reference prior for* $(\theta_1, \theta_2, \theta_3)$ *is*

$$\pi_R(\theta_1, \theta_2, \theta_3) \propto \theta_1^{-1}\theta_2^{-1}[mnp(log\theta_1)^2 + (m+n)^2\gamma_*]^{-\frac{1}{2}}. \quad (2.2)$$

Also, from Tibshirani(1989), we have matching priors for θ_1 *, the parameter of interest, as follows :*

Theorem 2.3. *Matching priors for* θ_1 *is given by*

$$\pi_M(\theta_1, \theta_2, \theta_3) \propto \theta_1^{-1}[mnp(log\theta_1)^2 + (m+n)^2\gamma_*]^{-\frac{1}{2}} g(\theta_2.\theta_3) \quad (2.3)$$

for any positive differentiable function g.

Bayesian inference for the system reliability R with known p is based on the posterior distributions under noninformative priors (2.1), (2.2), and (2.3).

REFERENCES

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RESUME

Ce article foye exclusivement en bayesiennes dédution pour la fiabilité d'un système multicompsant stress-puissance où les stress et puissance tous suivaient distribution gamma géneralement. Nous première dérivous a priori de Jeffrey, a priori de référénce et a priori de apparié pour un application paramétrique de l'interêt et alors nous analysons le distribution posteriori sous les priori de non-informatifs afine de estimater la fiabilité d'un système.