Logarithm as a Measure of Level for Development Type Index

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Abstract:
This paper interprets logarithm value as a measure of level for a variable and explores this basic meaning in producing composite measures by aggregating development variables that differ greatly in terms of nature, magnitude and unit of measurement. The paper terms the proposed methodology as Level Measure Index (LMI) and demonstrates its distinct merits against Malaysian Quality of Life Index (MQoLI) and Human Development Index (modified) methodologies that are currently in practice.

1. INTRODUCTION

The paper aims to illustrate a concept and methodology how log[1] value of a variable can be construed to provide a meaning as "a measure of level". The paper postulates that the measure of level meaningfully can be used to compile composite index instead of using actual indicators in measuring societal progress and quality of life.

2. STUDY CONTEXT

National statistical agencies and central planning organizations as well as international organizations are constantly taking great initiatives to publish various types of composite measures, besides publishing basic statistics and summary indicators[2]. Traditionally, the composite measures published are of economic [3] interest. Economic measures are aggregated meaningfully using monetary value as common measurement denominator for aggregating different types of economic variables. Attempt to use the economic approach in developing development or social type composite indexes[4] have been proven impractical, as social measures greatly differ in meaning, nature, magnitude and units of measurement. This paper proposes Level Measure Index for social type indexes and compares against traditional Laspeyres index, *modified* Human Development Index (HDI), Malaysian Quality of Life Index (MQoLI) methodologies.

3. MEASUREMENT METHODOLOGY AND RESULTS

3.1 The proposed LMI employs the following methodology: -

Additive Approach:

$$\mathbf{I_d} = \left[\frac{1}{k} \left\{ \sum \left[\log(V_t) / \log(V_0) \right] \right\} \right] * 100 \quad \text{or} \quad \mathbf{I_d} = \left[\sqrt[k]{\prod \{\log(v_t) / \log(V_0)\}} \right] * 100$$

Multiplicative Approach

where,

I_d is index value at a domain level; V_t is the current variable value at time t,; V_0 is the base variable value; Log(V_t) is the measure of level at period (t); Log(V_0) is the measure of level a variable referring to base period (o)

3.2 The table below illustrates the workability of the proposed method using fictitious numbers for the period T_1 to T_5.

Year >>>>	T_1	T_2	T_3	T_4	T_5
Variable Value, V_i	120	144	172.8	207.4	259.3
Annual change (%)		*20%*	*20%*	*20%*	*25%*
Laspeyres Type Index (LI)	100.0	120.0	144.0	172.8	207.4
MQoLI[5]	100.0	104.4	109.6	116.0	125.4
Consecutive change		*4.4*	*5.2*	*6.4*	*9.4*

[1] *Technically, log of a number is defined as the exponent power to a given base. As such, the term "power" can be interpreted as "level". Thus, when an exponent power changes it can be construed for changes in the level over time*

[2] *The raw numbers and summary indicators include measures such as number of live births, value of total exports, life expectancy, sex ratio, percentage or proportion of female participation in labour force compared to male counterparts et cetera*

[3] *Namely, Consumer Price Index (CPI), Producer Price Index (PPI), Industrial Production Index (IPI), Trade Index (TI), Stock Index (SI) using Laspeyres Index and Paasche Index methodology*

[4] *HumanDevelopment Index, Quality of life Index, Information Society Index et cetra*

[5] $I = [\{ (Y_i - Y_0) / \sigma \} * 10] + 100$ - *Y_i –current value; Y_o – base value; σ- standard deviation measure*

[6]HDI $_{Modified}$	100.0	100.17	100.38	100.63	101.0
Consecutive change	*0.17*	*0.21*	*0.25*	*0.37*	

Proposed Method					
Exponent Form, 10^{z}_{i}	$10^{2.0792}$	$10^{2.1584}$	$10^{2.2375}$	$10^{2.3167}$	$10^{2.4137}$
Measure of level	2.0792	2.1584	2.2375	2.3167	2.4137
LMI	100.0	103.8	107.6	111.4	116.1
Consecutive change in level	***3.8***	***3.8***	***3.8***	***4.7***	

The above computational procedure and practical results illustrates the following:-

i) The movement of the index number from period T_1 to T_5 under the proposed LMI method from 100.0 to 116.1 is appropriately reflects *gradual societal progress* compared to drastic movement from 100 to 207.4 indicated in the traditional index over the same period; QoLI also registers higher movement from 100 to 125.4; HDI $_{modified}$ records insignificant movement, understating the progress level.

ii) As shown in the illustration, the change in the proposed index value by 3.8 is equivalent to 20% change in traditional index value for any two consecutive periods from T1 to T4. Similarly, change in value by 4.7 in the proposed method proportionately reflected 25% rise in the traditional method. In other words, LMI merely does data transformation without altering its implicit meaning depicted in the data set. But, the QoLI and HDI $_{modified}$ fail to demonstrate such meaning, indicating inherent weakness in the computation.

iii) The QoLI is very sensitive to magnitude of standard deviation measure, σ which is a divisor in the formula. The σ measure is computed using past data and for many development variables the variation measure is small and hardly signifies societal changes. Moreover, the QoLI will become undefined when σ assumes zero value which is possible in practice when the variable value remain same over the study period or reduce to zero when the current value equals to the base value. While, HDI $_{modified}$ hardly moves because the range measure in the denominator is very large compared to range measure in the numerator. Similarly, HDI measure also can reduce to zero when actual value equals minimum measure.

iv) Table in the appendix provides LMI and QoLI measures using real life data and it can be seen the LMI is appropriately sensitive to the trend ingrained in the original data set especially reflected in crude death rate and number of people per doctor indicators[7].

4. CONCLUSION

Conclusively, the *level measure* employed in LMI provides a '*philosophical'* meaning for measuring development type measures. The log measure by its inherent property minimizes great variation present in the original measures; standardizes all variables to a common base, say base 10, irrespective of nature of the variable and converts a variable measure into a pure number by eliminating the unit of measurement - pure numbers can be treated algebraically. LMI is independent of past series. The numerator and denominator in LMI are in comparable in magnitude and will reduce meaningfully to one when the current and base variables are same. In practice, the LMI works well as development type variables do not assume zero or negative values[8]; obsolete[9] variables that are of no strategic importance can be dropped or can be replaced by new variables without affecting the validity of time series when sufficiently a large number of variables (say n items) are used in the index compilation - each variable algebraically assumes an equal weight of 1/n.

Reference:

- *Harry G. Costis (1972) Statistics For Business, pg 69-70, Charles E.Merill Publishing Co.,*
- *Human Development Report 1999 &2000, Published for the UNDP*
- *Malaysian Quality of Life, 99, Economic Planning Unit, Prime Ministers Department*
- *Official Statistical Publications of Department of Statistics*

[6] *HDI = (Actual Xi Value – Minimum Xi Value) / (Maximum Xi value – Minimum Xi value); Human Development Report uses this formula to compare HDI measures among member countries at a point of time; the measures also compared over time. For comparison purposes, this paper uses HDI modified version as:: HDI $_{modified}$ = HDI + 100*

[7] *The crude death rate registered overall decline from 4.7 deaths per 1000 in 1990 to 4.4 deaths per 1000 in 1999 Number of people per doctor showed a drastic improvement when the original figures moved from 2076 in 1996 to 1521 in 1997.*

[8] *Log zero and log of negative is undefined*

[9] *For example, the production of monochrome television can be replaced by colour television, if one compiles technology development index.*