

# Diffusion Smoothing in Brain Imaging

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Gaussian kernel smoothing has been widely used in smoothing 2D or 3D images such as magnetic resonance imaging (MRI), positron emission tomography (PET). An integral version of isotropic Gaussian kernel smoothing of the function  $f(\mathbf{x})$ ,  $\mathbf{x} = (x_1, \dots, x_n) \in \mathfrak{R}^n$  with smoothing parameter  $h > 0$  is defined by

$$F^*(\mathbf{x}, h) = \int_{\mathfrak{R}^n} K\left(\frac{\mathbf{x} - \mathbf{y}}{h}\right) \frac{f(\mathbf{y})}{h^n} d\mathbf{y}, \quad (1)$$

where the Gaussian kernel  $K(\mathbf{x}) = (2\pi)^{-n/2} \exp(-\|\mathbf{x}\|^2/2)$ . The formulation (1) does not work on curved surfaces such as the triangular surface mesh of the human brain (MacDonald *et al.*, 2000). However, by reformulating Gaussian kernel smoothing as a solution to a diffusion equation on a Riemannian manifold, the smoothing method can be generalized to an arbitrary curved surface. This generalization is called *diffusion smoothing* and has been used in the analysis of functional magnetic resonance imaging (fMRI) data on the brain surface (Andrade *et al.*, 2001) and detecting the regions of surface area change in brain development (Chung, 2001). Let  $F(\mathbf{x}, t) = F^*(\mathbf{x}, \sqrt{2t})$ . Then  $F(\mathbf{x}, t)$  is known to satisfy the diffusion equation

$$\frac{\partial F}{\partial t} = \Delta F, \quad F(\mathbf{x}, 0) = f(\mathbf{x}), \quad (2)$$

where  $\Delta = \frac{\partial^2}{\partial x_1^2} + \dots + \frac{\partial^2}{\partial x_n^2}$  is the Laplacian in  $\mathfrak{R}^n$ .

The generalization of the Laplacian  $\Delta$  to a Riemannian manifold  $M$  is called the *Laplace-Beltrami operator*. For the Riemannian metric  $ds^2 = \sum_{i,j=1}^n g_{ij} du^i du^j$  on  $M$ , the Laplace-Beltrami operator  $\Delta$  is defined as

$$\Delta F = \frac{1}{|g|^{1/2}} \sum_{i,j=1}^n \frac{\partial}{\partial u^i} \left( |g|^{1/2} g^{ij} \frac{\partial F}{\partial u^j} \right), \quad (3)$$

where  $|g| = \det(g_{ij})$  and  $g^{-1} = (g^{ij})$ . Using the finite element method, we estimate (3) on the triangular mesh of the brain surfaces (Chung, 2001). Let  $F(\mathbf{p}_i)$  be the data on the  $i$ -th node  $\mathbf{p}_i$  in the triangular mesh. If  $\mathbf{p}_1, \dots, \mathbf{p}_m$  are  $m$ -neighboring nodes around the central node  $\mathbf{p}$ , the Laplace-Beltrami operator is estimated as  $\widehat{\Delta F}(\mathbf{p}) = \sum_{i=1}^m w_i (F(\mathbf{p}_i) - F(\mathbf{p}))$  with the weights  $w_i = (\cot \theta_i + \cot \phi_i)/|T|$ , where  $\theta_i$  and  $\phi_i$  are the two angles opposite to the edge connecting  $\mathbf{p}_i$  and  $\mathbf{p}$ , and  $|T|$  is the sum of the areas of the  $m$ -incident triangles at  $\mathbf{p}$ . Afterwards, (2) is solved via the finite difference scheme:

$$F(\mathbf{p}, t_{n+1}) = F(\mathbf{p}, t_n) + (t_{n+1} - t_n) \widehat{\Delta F}(\mathbf{p}, t_n)$$

with the initial condition  $F(\mathbf{p}, t_0) = f(\mathbf{p})$  for each node  $\mathbf{p}$  on the triangular mesh. After  $N$ -iterations, the diffusion smoothing is locally equivalent to Gaussian kernel smoothing with smoothing parameter  $h = \sqrt{2}(t_N - t_0)^{1/2}$ .

We used this technique to enhance the sulcal patterns of the brain surfaces extracted from MRI (Chung, 2001). The maximum mean curvature can be used to characterize sulci, which is the hollows of the brain surface. The diffusion was run directly on the brain surface and mapped onto an ellipsoid later to show hidden sulci. **(a)** shows the mean curvature of the brain surface before the smoothing. **(b)** is after 40-iterations with fixed  $t_{n+1} - t_n = 0.02$ . **(c)** is after 100-iterations.

## REFERENCES

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