

An Improved CML Estimation Procedure for the Rasch Model with Item Response Data

Xiaoming Sheng

University of Alberta, Department of Mathematical Sciences,

632 CAB

Edmonton, AB T6G 2G1, Canada

xiaoming@stat.ualberta.ca

K. C. Carrière

University of Alberta, Department of Mathematical Sciences,

632 CAB

Edmonton, AB T6G 2G1, Canada

kccarrie@ualberta.ca

1. Introduction

This paper deals with the modelling of ordinal categorical data: binary (“yes” or “no”) data, Poisson counts (number of successes), rating scales (“disagree,” “neutral,” and “agree,” etc.), partial credit (awarding partial credit on intermediate levels of performance), and other ordinal categorical data.

We consider modifying the CON procedure to implement a simultaneous estimation method for the polychotomous Rasch model, based on the conditional maximum likelihood. We also derive the asymptotic properties of the CML estimators and then develop a conditional likelihood ratio test for the goodness-of-fit of the model.

2. Properties of the New Estimators

CML estimators are consistent under regularity conditions when a minimal sufficient statistic exists.

Theorem 1:

1. When $N(r) \rightarrow \infty$, then $\hat{\delta}_{\mathbf{x}_j \in \mathcal{R}} \xrightarrow{P} \delta_0$, and furthermore $\hat{\delta}_{\mathbf{x}_j \in \mathcal{R}}$ is asymptotically normally distributed, with mean δ_0 and asymptotic covariance matrix $H^{-1}(r, \delta_0)/N(r)$.
2. When $N(r) \rightarrow \infty$ for all r , then $\hat{\delta} \xrightarrow{P} \delta_0$, i.e., the overall CML estimators for δ are consistent, and $\hat{\delta}$ is also asymptotically normally distributed, with mean δ_0 and asymptotical covariance matrix $H^{-1}(\delta_0)$.

For a sketch of the proof, please see Sheng and Carrière (2001).

3. Simulation Results

Figure I: Q-Q plot of G^2 vs quantiles of the χ^2 distribution

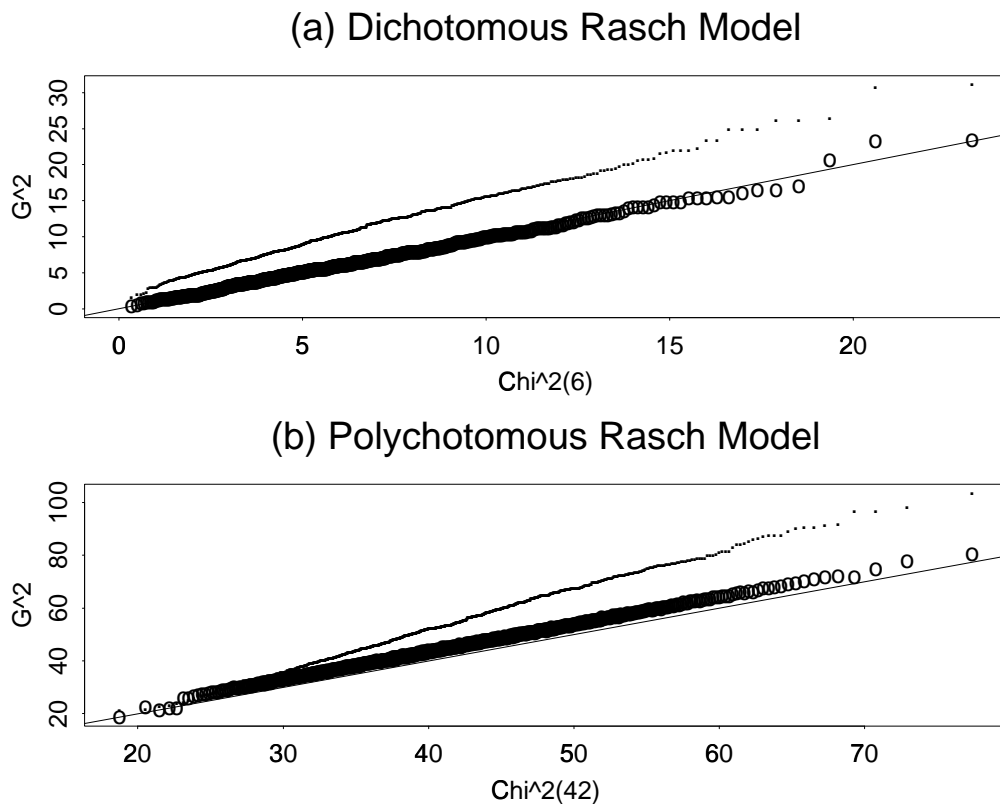


Figure I shows the Q-Q plots of G^2 versus quantiles from the χ_6^2 (Figure 1a) and χ_{42}^2 (Figure 1b) distributions, using our modified approach and the conventional CML approach, respectively. The plot clearly shows that our G^2 is approximately distributed as the assumed χ^2 .

REFERENCES

Sheng, X., and Carrière, K.C. (2001). Technical Paper Series No.01-01. Department of Mathematical Sciences, University of Alberta.

RESUME

Le but primaire dans modeler des donnees de rponse d'lment est de trouver une seule mesure des capacits de la personne et des difficults d'lment qui satisfait les propriets de la mesure fondamentale. En cet article, bas sur le maximum de vraisemblance conditionnel, nous avons appliqu une mthode simultane d'valuation qui peut comparer les paramtres de Recht plus efficacement. Notre mthode d'valuation surpasse CON dans l'ajustement de modle et la prcision des estimateurs de Recht.