Maximum Likelihood Predictive Densities for the Length-biased Inverse Gaussian Distribution

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1. SUMMARY

This article deals with the estimation of parameters of a length-biased inverse Gaussian distribution via maximum predictive density estimation.

2. INTRODUCTION

A length-biased distribution arises as a selection model where the sampling distribution follows a weighted version of the original population due to a stochastic mechanism which distorts the probability of an item being selected. When observations are selected with probability proportional to their "length", the resulting distribution is called *length-biased.* A distribution function G_F defined on R^+ is called *length-biased* distribution corresponding to a df F (also defined on R^+), if

$$G_F(y) = \mu_F^{-1} \left\{ \int_0^y x \cdot dF(x) \right\}, \ \ \forall y \in R^+,$$

where $\mu_F = \int_0^\infty x \cdot dF(x)$

Thus, the length-biased pdf can be written as

$$g(x) = \frac{x \cdot f(x)}{E(X)}$$

Predictive Density Function

Let $X_1, \ldots, X_n$ be a sample of past observations, and let T be a future observation from a population with pdf $f(x;\theta)$. Then the MLPD for $T = t$ based on $\underline{X} = \underline{x}$ defined as

$$\hat{f}(t|x) \propto max_\theta f(t;\theta) f(x;\theta).$$

In the case of length-biased inverse Gaussian random variable, the joint pdf

$$\begin{aligned} f(t;\mu,\lambda)f(x;\mu,\lambda) &= (\frac{\lambda}{2\pi\mu^2})^{\frac{(n+1)}{2}} t^{-\frac{1}{2}} \prod_{i=1}^{n} x_i^{-\frac{1}{2}} \exp\{-\frac{\lambda}{2\mu^2}\sum\frac{(x_i-\mu)^2}{x_i}\} \\ &\cdot \exp\{-\frac{\lambda}{2\mu^2}\frac{(t-\mu)^2}{t}\} \end{aligned} \quad (1)$$

The maximum of the joint pdf with respect to μ, or λ (or w.r.t. both) provides us with (restricted or unrestricted) MLPD's for t.

Estimation of Reliability

The reliability function is defined as $R(t_0) = 1 - F(t_0)$ where $F(x) = \int_0^x f(y)dy$. The reliability function of LBIG is given by

$$\Phi(-\alpha(t_0)) + \exp(2\lambda/\mu)\Phi(\beta(t_0))$$

where $\alpha(t_0) = \sqrt{\lambda/t_0}(t_0/\mu - 1)$, $\beta(t_0) = -\sqrt{\lambda/t_0}(t_0/\mu + 1)$, and $\Phi(.)$ is the cdf of standard normal random variable. In the sense of maximum likelihood estimation, one would replace the parameters with their MLE's to obtain an estimate of the reliability at a fixed time point. In the case of MLPD estimation, we proceed by replacing the pdf with its MLPD estimate to obtain an estimate of the reliability. The MLPD of $R(\tilde{t}|\theta) = 1 - \int_0^t \tilde{f}(y|x)dy$ where $f\tilde{(.)}$ is an MLPD of $f(.)$.

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RESUME

Cet article traite l'valuation des paramtres d'une distribution gaussienne inverse longueur-dcentre par l'intermdiaire d'valuation prdictive maximum de densit.