A Simple Technique for Drawing Simulation Samples in State Space Time Series Analysis

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We shall consider a standard linear Gaussian state space model. Such models provide the basis for the study of a broad class of problems in time series analysis; in particular, Box-Jenkins ARIMA models can be regarded as special cases of models of this type. In state space models, the development of the system over time is represented by a sequence of unobserved vectors a(t), called the state vectors, and associated with these is a sequence of observations y(t) for t=1,⋯,n. The relation between a(t) and y(t) is equivalent to a linear regression of y(t) on a row vector z(t) with coefficient vector a(t) which varies over time according to a Gaussian first-order vector autoregression. We denote the disturbances in the regression model by b(t) and the disturbance vectors in the autoregression by c(t). An important problem in state space time series analysis is the drawing of random samples for simulation purposes from the conditional distributions of the disturbances b(t) and c(t) given the observations. Such samples are required for the analysis of non-Gaussian and nonlinear models for reasons spelt out in Durbin and Koopman (2000) and Durbin and Koopman (2001).

The standard technique for drawing these samples at present is to use the simulation smoother of de Jong and Shephard (1995). Taking first the disturbance b(t), these authors provide a backwards recursion for drawing a sample value from the conditional distribution of b(t) given y(1), ⋯y(n) and also given b(t+1),⋯,b(n), for t=n,n-1,⋯,1. Formulae for this recursion and the analogous recursion for disturbance c(t) will be given in the lecture.

This paper presents a new solution for the problem which is simpler and computationally more efficient than that of de Jong and Shephard. It is based on the simple observation that the variance matrix of a vector of values in a multivariate normal distribution, given that a second vector of values is held fixed, does not depend on the second vector. Let B(t) be the conditional expectation of b(t) given y(1),⋯,y(n). This is easy to compute using a standard smoothing recursion as described in Durbin and Koopman (2001). Now draw a random sample b*(1),⋯,b*(n),c*(1), ⋯,c*(n) from the unconditional distribution of b(1),⋯,b(n),c(1),⋯,c(n) and substitute these in the model to obtain a simulated sample of values y*(1),⋯,y*(n) of y(1),⋯y(n). Let B*(t) be the conditional expectation of b*(t) given y*(1),⋯,y*(n) which we compute by the smoothing recursion. Let b**(t) = B(t) + b*(t) - B*(t) for t = 1,⋯,n. It follows from the above simple observation that b**(1),⋯,b**(n) have the required conditional distribution of b(1),⋯,b(n) given y(1),⋯,y(n). The process can be repeated as often as required. Similar results hold for c(1),⋯,c(n). Details are given in Durbin and Koopman (2001*).

Apart from its intrinsic simplicity, this technique is easier to incorporate in new software than the technique of de Jong and Shephard since it only requires standard Kalman filter and smoother routines.

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RESUME

Nous decrivons une nouvelle methode pour simuler un modele pour un serie chronologique quand les observations sont fixees.