Estimation for Nonlinear Autoregressive Models Generated by Beta-ARCH Processes

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1. Introduction

In a seminal paper, Engle(1982) introduced conditional heteroscedastic autoregressive(ARCH) models in which the conditional variance(or volatility) was specified as a linear combination of the squared residuals. As indicated in Engle(1982), the conditional variance can be modeled using other functional forms rather than squared residuals. In order to take care of the richer class of conditional variances compared to the classical ARCH processes, Guegan and Diebolt(1994) suggested the first order beta-ARCH process and this was extended to the m-order cases by An et al.(1997).

An et al.(1997) investigated probabilistic structures such as ergodicity and existence of stationary moments for the m-order beta-ARCH process. Regarding estimation, Hili(1999) derived minimum Hellinger distance estimators of the parameters for the first order beta-ARCH models. To our knowledge, limiting results concerning the parameter estimation for beta -ARCH has not yet been adequately spelled out in the liturature except Hili(1999).

In the present paper we are concerned with nonlinear autoregressive processes generated by beta-ARCH errors. This article is mainly addressing the problem of parameter estimation and derives the relevant large sample properties of the estimators. Specifically we will discuss least squares as well as maximum likelihood estimation for the model. Non-differentiability problems arising from the special structure of the model are circumvented via iterative methods in least squares estimation and can be resolved by employing *Frechet* (almost sure) derivatives(cf. Bickel et al.(1993), appendix) in deriving efficient one-step maximum likelihood estimators. A simple sufficient condition for the ergodicity of model is discussed. For parameter estimation we consider least squares estimation and also discuss the local asymptotic normality(LAN) property of the log-likelihood ratio using the quadratic mean differentiablity approach, from which a class of (asymptotically efficient) one-step maximum likelihood estimators of parameters can be deduced.

2. Main Results

A set of sufficient conditions ensuring that the model is geometrically ergodic is derived via the standard arguments as in Feigin and Tweedie(1985). The question as to whether the stationary moments exist up to certain order is briefly discussed. Due to the structure of the objective function from which the least squares estimators are obtained via minimization, two difficulties arises : nonlinearity and non-differentiability. Under some regularity conditions plus the finite sixth order moment condition, the iterative least squares estimators are shown to be asymptotically normal.

The problem for the asymptotic expansion of log-likelihood ratio and related one-step maximum likelihood estimator(MLE) is studied. To circumvent technical difficulties of non-differentiability, we employ the arguments of Frechet differentiability which is also referred to as the differentiability in quadratic mean. See Bickel et al. (1993, appendix) for the comprehensive treatments of the quadratic mean derivatives. Under broad conditions it is obtained that the log-likelihood ratio possesses the local asymptotic normality(LAN) property, from which a class of one-step MLE is constructed. The null and non-null limiting distributions of the MLE are also given.

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