Surprising Effects of a Trended Regressor on the Efficiency Properties of the Least Squares and Stein-Rule Estimation of Regression Coefficient

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The least squares method possesses the celebrated property of providing the optimal estimator for the coefficient vector in a linear regression model in the class of linear and unbiased estimators. If we take the performance criterion as risk under a quadratic loss function, James and Stein (1961) have demonstrated that it is possible to find nonlinear and biased estimators with smaller risk in comparison to the least squares estimator. This pioneering result has led to the development of several families of estimators having superior performance under risk criterion. Among them, the Stein-rule family characterized by a single scalar has acquired considerable popularity and importance, see, e.g., Judge and Bock and Judge, Griffiths, Hill, Lütkepohl and Lee (1985).

The properties of Stein-rule estimators have been extensively studied in the literature but most of the investigations particularly dealing with large sample properties have been conducted under the specification that the regressors in the model are asymptotically cooperative, i.e., the limiting form of the variance covariance matrix of the explanatory variables as the number of observations tends to infinity is a finite and nonsingular matrix. Such an assumption may be violated in many practical situations, for instance, where some regressors are trended. In particular, when one of the explanatory variables has a linear or more generally polynomial trend, its variance tends to infinity and consequently the limiting form of the variance covariance matrix of the explanatory variables is no more finite. Similarly, if the variable follows an exponential trend, its variance tends to zero and thus the limiting form of the variance covariance matrix of the explanatory variables becomes a singular matrix.

The following linear regression model is postulated:

$$y = ae + Xb + dZ + e$$

where y is a $n\times1$ vector of n observation on the study variable, a is a scalar representing the intercept term in the regression relationship, e is a $n\times1$ vector with all elements unity, X is a $n\times p$ matrix of n observations on p explanatory variables, b is a $p\times1$ vector of p regression coefficients, Z is a $n\times1$ vector of n observations on another explanatory variable, d is the regression coefficient associated with it and e is a $n\times1$ vector of disturbances.

It is assumed that the elements of vector e are independently and identically distributed following a normal distribution with mean 0 and finite but unknown variance s^2.

If we define

$$A = I_n - \frac{1}{n}ee'$$

the ordinary least squares estimators least squares estimators b and d of b and d respectively are

$$d = \frac{Z'Ay - Z'AX(X'AX)^{-1}X'Ay}{Z'AZ - Z'AX(X'AX)^{-1}X'AZ}$$

$$b = (X¢AX)^{-1}X¢A(y - dZ)$$

whereas the Stein-rule estimators of b and d are

$$\hat{b} = \left[1 - \left(\frac{k}{n}\right)\frac{(y - Xb - dZ)' A (y - Xb - dZ)}{(Xb + dZ)' A (Xb + dZ)}\right] b$$

$$\hat{d} = \left[1 - \left(\frac{k}{n}\right)\frac{(y - Xb - dZ)' A (y - Xb - dZ)}{(Xb + dZ)' A (Xb + dZ)}\right] d$$

where k is the nonstochastic scalar, independent of n, characterizing the scalar.

Assuming the elements of Z are trended and follow a

(a) linear trend specified by

$$Z_t = q_0 + q_1 t \qquad (t = 1, 2, \ldots, n)$$

(b) quadratic trend specified by

$$Z_t = q_0 + q_1 t + q_2 t^2 \qquad (t = 1, 2, \ldots, n)$$

(c) exponential trend specified by

$$Z_t = q_0 + q_1\, q_2^{t-1} \qquad (t = 1, 2, \ldots, n),$$

where q_0, q_1 and q_2 are the constants determining the nature of trend.

The efficiency properties of the ordinary least squares and Stein-rule estimators are derived using the large sample asymptotic approximation theory. The superiority of these estimators is discussed in the sense of Löewner ordering, i.e., with respect to the criterion of mean squared error matrix and dominance conditions are derived.

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