Test of Variance Change in a Long-Memory Process

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1. Introduction

Many econometric time series have long-memory properties. Since the long-memory can be observed by data obtained from rather a long period, there can be some change in parameter such as variance. We consider testing variance change in a long-memory process.

2. Test for the Change in Variance

Consider the variance change model in the following:

$$X_i = \begin{cases} \mu + \sigma u_i, & i = 1, \cdots, k^* \\ \mu + \theta u_i, & i = k^* + 1, \cdots, n, \end{cases} \tag{1}$$

where the error process $\{u_i\}$ is a stationary process with zero mean, unit variance and autocovariances satisfying

$$lim_{n\to\infty} \sum_{1\le i\le n} \sum_{1\le j\le n} \gamma(i-j)/(n^{2H}L(n)) = 1. \tag{2}$$

$H > 1/2$ implies a long-memory process. Following Gombay et al. (1996), the test of change in variance is based on the functional

$$M_n(\tau) = \frac{1}{\xi_0}[\sum_{1\le i\le [n\tau]} (X_i - \mu)^2 - \tau \sum_{1\le i\le n} (X_i - \mu)^2], \quad where\ \xi_0^2 = Var_{H_0}(X_1 - \mu)^2. \tag{3}$$

For the derivation of the test statistic, Giraitis and Taqqu (1999) showed that the limiting properties of the form

$$\frac{1}{\sqrt{A(n)}} \sum_{1\le i\le [n\tau]} G(X_i), \quad 0 < \tau < 1, \tag{4}$$

only depends on the Hermite rank of G if the process $\{X_i\}$ is a stationary Gaussian long-memory process and G is an arbitrary function, using Taqqu's strong reduction theorem (1975, 1979). Non-Gaussian case is similar to the Gaussian case and the limiting distribution of the test statistic can be derived as follows.

Theorem 1

For a strongly dependent process $\{X_i\}$ *satisfying (1) and (2) for* $0 < H < 1$*, assume that* $H_0 : \hat{k}^* > n$ *holds. Then there exist non-degenerate processes* $\bar{Z}_2^1(\tau)$ *and* $\bar{Z}_2^2(\tau)$ *such that*
(i) if $1/2 < H < 3/4$*, then*

$$M_n(\tau) \to \bar{Z}_2^1(\tau) \tag{5}$$

and
(ii) if $3/4 < H < 1$*, then*

$$M_n(\tau) \to \bar{Z}_2^2(\tau) \tag{6}$$

for all $0 < \tau < 1$*.*

Since the Gaussianity of the limit process and the limit process itself are dependent on the nuisance parameter H, we cannot obtain standardized critical regions for testing. But if we put

$$\delta = \sigma^2 - \theta^2 \tag{7}$$

and

$$g_n(\tau) = \begin{cases} -(n-k^*)\delta\tau, & 0 \le \tau \le k^*/n \\ -k^*\delta(1-\tau), & k^*/n \le \tau \le 1 \end{cases} \tag{8}$$

the consistency of the test statistic can be derived.

Theorem 2

For a strongly dependent process $\{X_i\}$ *satisfying (1) and (2) for* $0 < H < 1$*, there exist non-degenerate processes* $\bar{Z}_2^1(\tau)$ *and* $\bar{Z}_2^2(\tau)$ *such that we have*
(i) if $1/2 < H < 3/4$*, then*

$$\frac{1}{\xi_0}[\{\sum_{1\le i\le [n\tau]}(X_i-\mu)^2 - \tau\sum_{1\le i\le n}(X_i-\mu)^2\} - g_n(\tau)] \to \bar{Z}_2^1(\tau) \tag{9}$$

and
(ii) if $3/4 < H < 1$*, then*

$$\frac{1}{\xi_0}[\{\sum_{1\le i\le [n\tau]}(X_i-\mu)^2 - \tau\sum_{1\le i\le n}(X_i-\mu)^2\} - g_n(\tau)] \to \bar{Z}_2^2(\tau) \tag{10}$$

for all $0 < \tau < 1$ *under* H_A*.*

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