Test of Variance Change in a Long-Memory Process

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1. Introduction

Many econometric time series have long-memory properties. Since the long-memory can be observed by data obtained from rather a long period, there can be some change in parameter such as variance. We consider testing variance change in a long-memory process.

2. Test for the Change in Variance

Consider the variance change model in the following:

\[ X_i = \begin{cases} 
\mu + \sigma u_i, & i = 1, \ldots, k^* \\
\mu + \theta u_i, & i = k^* + 1, \ldots, n, 
\end{cases} \quad (1) \]

where the error process \( \{u_i\} \) is a stationary process with zero mean, unit variance and autocovariances satisfying

\[ \lim_{n \to \infty} \sum_{1 \leq i \leq n} \sum_{1 \leq j \leq n} \gamma(i - j)/(n^{2H}L(n)) = 1. \quad (2) \]

\( H > 1/2 \) implies a long-memory process. Following Gombay et al. (1996), the test of change in variance is based on the functional

\[ M_n(\tau) = \frac{1}{\xi_0^2} \left[ \sum_{1 \leq i \leq [\tau n]} (X_i - \mu)^2 - \tau \sum_{1 \leq i \leq n} (X_i - \mu)^2 \right], \quad \text{where } \xi_0^2 = \text{Var}_0(X_1 - \mu)^2. \quad (3) \]

For the derivation of the test statistic, Giraitis and Taqqu (1999) showed that the limiting properties of the form

\[ \frac{1}{\sqrt{A(n)}} \sum_{1 \leq i \leq [\tau n]} G(X_i), \quad 0 < \tau < 1, \quad (4) \]

only depends on the Hermite rank of \( G \) if the process \( \{X_i\} \) is a stationary Gaussian long-memory process and \( G \) is an arbitrary function, using Taqqu’s strong reduction theorem (1975, 1979). Non-Gaussian case is similar to the Gaussian case and the limiting distribution of the test statistic can be derived as follows.
Theorem 1

For a strongly dependent process \( \{X_i\} \) satisfying (1) and (2) for \( 0 < H < 1 \), assume that \( H_0 : \hat{k} > n \) holds. Then there exist non-degenerate processes \( \bar{Z}_1^1(\tau) \) and \( \bar{Z}_2^2(\tau) \) such that

(i) if \( 1/2 < H < 3/4 \), then

\[
M_n(\tau) \to \bar{Z}_1^1(\tau)
\]

and

(ii) if \( 3/4 < H < 1 \), then

\[
M_n(\tau) \to \bar{Z}_2^2(\tau)
\]

for all \( 0 < \tau < 1 \).

Since the Gaussianity of the limit process and the limit process itself are dependent on the nuisance parameter \( H \), we cannot obtain standardized critical regions for testing. But if we put

\[
\delta = \sigma^2 - \theta^2
\]

and

\[
g_n(\tau) = \begin{cases} 
-(n - k^*)\delta\tau, & 0 \leq \tau \leq k^*/n \\
-k^*\delta(1 - \tau), & k^*/n \leq \tau \leq 1
\end{cases}
\]

the consistency of the test statistic can be derived.

Theorem 2

For a strongly dependent process \( \{X_i\} \) satisfying (1) and (2) for \( 0 < H < 1 \), there exist non-degenerate processes \( \bar{Z}_1^1(\tau) \) and \( \bar{Z}_2^2(\tau) \) such that we have

(i) if \( 1/2 < H < 3/4 \), then

\[
\frac{1}{\xi_0}\left\{ \sum_{1 \leq i \leq \lfloor n\tau \rfloor} (X_i - \mu)^2 - \tau \sum_{1 \leq i \leq n} (X_i - \mu)^2 \right\} - g_n(\tau) \to \bar{Z}_1^1(\tau)
\]

and

(ii) if \( 3/4 < H < 1 \), then

\[
\frac{1}{\xi_0}\left\{ \sum_{1 \leq i \leq \lfloor n\tau \rfloor} (X_i - \mu)^2 - \tau \sum_{1 \leq i \leq n} (X_i - \mu)^2 \right\} - g_n(\tau) \to \bar{Z}_2^2(\tau)
\]

for all \( 0 < \tau < 1 \) under \( H_A \).

REFERENCES


