Order-Logit Map: Inferring Individual Preferences in a Product-Market Map from Preference Ordered Data

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1. Introduction

Internal analysis of consumer preferences infers brand positions in an attribute space from preference and choice data. This paper proposes Order-Logit Map, a model that infers individual preferences in a brand-positioning map from consumer ordered preference data. Order Logit Map is an ordered logit model, a random utility model and a multidimensional scaling procedure. In the estimation procedure, a Monte Carlo method is used to simulate the high order integrals that enter the corresponding likelihood function. For previous models for brand positioning methods, Choice map (Elrod(1988)) is a stochastic brand choice model, but based on panel data.

2. Model

U_{ij} denotes the utility of brand j for consumer i as:

$$U_{ij} = \beta_{i1}a_{j1} + \beta_{i2}a_{j2} + \alpha_i \log p_j + \epsilon_{ij}, \tag{1}$$

where p_j is the price of brand j, α_i is a price coefficient, $\mathbf{a}_j = (a_{j1}, a_{j2})$ are a position of brand j in M-dimensional map, and $\boldsymbol{\beta_i}= (\beta_{i1}, \beta_{i2})$ are the consumer's importance weights for these dimensions. $\mathbf{a}_j$ and $\boldsymbol{\beta_i}$ are unobservable by the analyst. The importance vector $\boldsymbol{\beta_i}$ is assumed, without loss of generality, to follow bivariate normal distribution with mean vector $\boldsymbol{\mu_i}= (\mu_{i1}, \mu_{i2})$ and an identity covariance matrix $\boldsymbol{I}$. Furthermore, assume that ϵ_{ij} in Equation (**??**) follows the standard double exponential distribution: $P[\epsilon \leq x] = \exp[\exp(-x)]$, and that ϵ_{ij} is independently and identically distributed over the brands. In this paper, the models are estimated based on preference ordered data. Respondents (or consumers) are asked to give a preference ordering for a set of brands in a product category, in which partial ordering is allowed. d_{ik} denotes the kth most preferred brand for respondents i, and J_i is the number of brands considered by respondent i, corresponding to a depth of ordering. This rank ordering is interpreted as a series of $(J_i - 1)$ choices of the most preferred brand d_{ik} out of the set of brands $D_i(k) = \{d_{ik}, d_{i(k+1)}, \cdots, d_{J_i}\}$, $k = 1, \cdots, J_i$. Then the probability of ordering D_i is specified as:

$$P[D_i, (\mathbf{a}_j, \cdots, \mathbf{a}_{J_i}), \boldsymbol{\mu_i}] = \prod_{k=1}^{J_i} \int_{-\infty}^{\infty} \frac{\exp(\beta_{i1}a_{d_{ik}1} + \beta_{i2}a_{d_{ik}2} + \alpha_i \log p_{d_{ik}})}{\sum_{d_{ig} \in D_i(k)} \exp(\beta_{i1}a_{d_{ig}1} + \beta_{i2}a_{d_{ig}2} + \alpha_i \log p_{d_{ig}})} z[\boldsymbol{\beta_i} - \boldsymbol{\mu_i}] d\boldsymbol{\beta_i}, \tag{2}$$

where $z[\cdot]$ denotes the prior density function for importance vector $\boldsymbol{\beta_i}$ (the standard bivariate normal distribution). Then, the probability for observing ordering (D_1, D_2, $\cdots$, D_N) given by N respondents is:

$$P[(D_1, D_2, \cdots, D_N), (\mathbf{a}_j, \cdots, \mathbf{a}_J), \boldsymbol{\mu_i}] = \prod_{i=1}^{N} P[D_i, (\mathbf{a}_j, \cdots, \mathbf{a}_{J_i}), \boldsymbol{\mu_i}]. \quad (3)$$

where J is the total number of brands.

3. Estimation

The optimization procedure searches for a maximum of the likelihood given by Equation (**??**) over the parameter $\mathbf{a}_j$ (the brand positions), $\boldsymbol{\mu_i}$ (the importance weights) and α_i (the price coefficient). Here, Monte Carlo methods are used to the high order integrals that enter the likelihood function. The function actually maximized over these parameters is:

$$P[(D_1, D_2, \cdots, D_N), (\mathbf{a}_j, \cdots, \mathbf{a}_{J_i}), \boldsymbol{\mu_i}] \quad (4)$$

$$\approx \prod_{i=1}^{N} \prod_{k=1}^{J_i} \frac{1}{L} \sum_{l=1}^{L} \frac{\exp\{(\mu_{i1} + z_{i1}^{(l)})a_{d_{ik}1} + (\mu_{i2} + z_{i2}^{(l)})a_{d_{ik}2} + \alpha_i \log p_j\}}{\sum_{d_{ig} \in D_i(k)} \exp\{(\mu_{i1} + z_{i1}^{(l)})a_{d_{ig}1} + (\mu_{i2} + z_{i2}^{(l)})a_{d_{ig}2}\}} \quad (5)$$

where $\mathbf{z}_i^{(l)} = (z_{i1}^{(l)}, z_{i2}^{(l)})$, $l = 1, 2, \cdots, L$ are random vectors drawn from the standard bivariate normal distribution $N(\mathbf{0}, \boldsymbol{I})$ and L is the number of draws.

4. Example

The applicability of Order-Logit Map is tested on an actual data set: preference for personal computer. Table 1 shows the parameter estimates.

Table 1. Prameter Estimates

(a) Estimates for Price Coefficient α_i and Importance Vectors (μ_{i1}, μ_{i2})

	Respondent									
	1	2	3	4	5	6	7	8	9	10
α_i	0.18	-0.34	0.03	0.23	0.01	0.28	-0.35	0.30	-0.21	0.12
μ_{i1}	1.11	-1.21	-1.08	1.31	-0.19	1.29	1.15	0.22	1.15	0.99
μ_{i2}	-0.20	-0.25	0.71	-0.98	0.15	-0.48	0.32	-0.88	-1.35	-0.35

(a) Estimates for Brand Positions (a_{1j}, a_{2j})

	Brand for Personal Computer									
	VAIO	FM V	Value Star	Aptiva	Mebius	Prius	Performance	PC Station	Presario	iMac DV+
a_{j1}	0.29	-0.43	0.02	-0.08	0.00	0.08	-0.11	-0.56	-0.22	-0.58
a_{j2}	0.08	-0.13	-0.45	-0.52	0.00	0.26	-0.30	0.56	0.13	0.12

REFERENCES

Elrod, T.(1988). Choice Map: Inferring a Product-Market Map from Panel Data. *Marketing Science*, **7**, 21-40.