Bayesian methods in actuarial science: A perspective

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1. Abstract

Bayesian ideas were introduced into actuarial science from the late 1960s in the form of empirical credibility methods for premium setting. The advance of the Bayesian methodology was slow due to its subjective nature and due to the computational difficulties associated with the full Bayesian analysis. The paper offers a brief survey of Bayesian solutions to some actuarial problems and discusses the current state of research

2. Introduction

Bayesian methodology is used in various areas within actuarial science. This paper does not aim at providing an extensive review of all applications of this methodology, but rather seeks to point out the main areas of application and to identify the main themes of contemporary research.

The most important areas of application are Experience rating (including credibility theory), where premiums are set, given the accumulated past claims in a portfolio, and loss reserving, that deals with the amount of reserves to be kept by an insurance company.

An account of Bayesian statistics in actuarial science can be found in a book by Klugman (1992) which concentrates on the Bayesian approach to credibility. For recent review papers see Makov *et al.* (1996) and Schmidt (1998).

3. Theoretical and computational issues: the case of experience rating

In experience rating θ_{ij} $(i = 1,...I, j = 1,...J)$, the *risk parameter*, is regarded as the total characterization of the risk of contract i at time j. The *fair premium* is denoted by $\mu(\theta_{ij}) = E[X_{ij}|\theta_{ij}]$ and the actual premium is calculated by $E[X_{i,J+1}|D] = E[\mu(\theta_{ij})|D]$, where $D = X_{ij},(i = 1,...I, j = 1,...J)$. Since $E[\mu(\theta_{ij})|D]$, the *exact credibility,* was typically hard to calculate, empirical Bayes techniques were employed to produce an estimate of the exact credibility by means of the famous credibility formula $zm + (1-z)\bar{x}_i$, where m is the mean of $U(.)$ and $\bar{x}_i$ the mean claim of the ith contract. z, the credibility factor, is chosen to produce the best (m.s.e) linear empirical bayes estimator of the exact credibility.

The empirical Bayes credibility model (Bűhlman, 1967,1969 and Bűhlman and Straub, 1970) was a successful practical compromise at a time when opposition to the Bayesian methodology was centered on two major points: **a**. opposition to the subjective nature of Bayesian statistics and the search for a more objective tool. **b**. reservations about its applicability, especially as closed-form analytical solutions were not widely available.

The traditional credibility formula, while being a distribution free method, was shown (Jewell, 1974 and others) to produce an estinate equal to the Bayesian estimate for a large class of problems (exponential family/conjugate priors). Landsman and Makov (1998,1999a) established that the simple credibility formula is also correct, in a Bayesian sense, for claim distributions belonging to the Exponential Dispersion Family (EDF), thus allowing computationally simple estimation of the fair premium for members of this family. Landsman and Makov (1999b,1999c) established approximations suitable for distributions (like the Log-Gamma and Log-Normal) which are outside these families.

The issue of prior specification has recently been dealt with in various ways. Young (1997) suggested using kernel density estimation to estimate the prior distribution of the parameter of interest. See also Young (2000). Information measures were used to establish a prior distribution for the dispersion parameter λ of the EDF (Landsman and Makov, 1998, 1999a, 2001). Gómez-Déniz *et al.* (1999) carried out robustness analysis with respect to the prior distribution by considering a contaminated class of prior distribution.

For many years the implementation of Bayesian models was only possible for simple low dimensional problems (see Klugman, 1992, for analytical approximations suitable for such cases). During the last decade, however, there has been an increasing realization that the computations required for full Bayesian analysis can be carried out effectively by means of simulation based methods and, in particular, Markov chain Monte Carlo (MCMC) methods (Gilks *et al.*, 1996). In effect, the experience rating problem discussed above can be fully investigated on a PC by means of an hierarchical Bayes model for a portfolio as large as needed. For an introduction to MCMC and their actuarial applications see Makov *et al.* (1996) and Scollnik (1996).

4. Loss reserving

Loss reserves are needed whenever losses remain unpaid at the end of a year, typically as a result of claims Incurred But Not yet Reported (IBNR) or claims Reported But Not Settled (RBNS).

Let the random variables X_{ij} $(i = 1, ..., I; j = 1, ...I - i + 1)$ denote claim figures (or loss ratios, claim frequencies, etc.) for the i^{th} year of origin at the j^{th} development year. The data constitute a triangle (the so called *run-off triangle),* where the upper triangle is given and the lower triangle is to be estimated. Since the problem is crucial for insurance companies, a lot of research has been invested in developing effective methodologies (Taylor, 2000). However, relatively little was done from the Bayesian perspective. Verrall (1990) analyzed the chain ladder model, which can be interpreted as the two way model: $\log(X_{ij}) = \mu + \alpha_I + \beta_J + \varepsilon_{ij}$, but no attempt was made to treat the unknown variances in a fully Bayesian manner. A complete hierarchical Bayes model was implemented by Hazan and Makov (2000), where MCMC was used to estimate the parameters of two models: the chain ladder model and a switching regression model which allows the delayed claims to increase up to a point and then decrease over time. MCMC was also employed in Ntzoufras and Dellaportas (2001) where various models for outstanding claims problem are discussed and were claim count uncertainty is incoroporated.

The benefit of the Bayesian approach is in providing the decision maker with a posterior predictive distribution for every entry in the lower triangle of the run-off triangle. Such a distribution allows the assessment of the required reserves in terms not only of point estimators but also of Bayesian confidence intervals and probabilistic indication on the chance that the amount of a future claim exceeds a given threshold.

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RESUME

Les idées bayesiennes pénètrent dans les sciences actuarielles à partir des annees soixante du siècle passé sous la forme des méthodes empiriques de crédibilité de fixation de primes. Les progrès de la méthode bayesienne furent lents à cause de leur nature subjective et leurs difficultés de calcul, dues à l'analyse complète bayesienne. Notre communication est une brève préséntation des solutions bayesiennes de quelques problèmes actuariels et de l'état actuel de la recherche sur ces questions.