

Hybrid distributions

Rose D Baker

University of Salford, School of Accounting, Economics and Management Science

The Crescent

Salford M5 4WT, UK

r.d.baker@dial.pipex.com

1. Introduction

By a hybrid distribution is meant one whose pdf changes functional form at one or more cutpoints, such that in the simplest case, the pdf $f(x)$ is given by:

$$f(x) = \begin{cases} c_1 g(x) & \text{if } x < a \\ c_2 h(x) & \text{if } x > a, \end{cases}$$

where g and h are pdfs. If $c_1 = 1$, the hazard function changes from the hazard function of g to that of h at $x = a$.

There are few examples of such distributions in the statistical literature, such as the construction of piecewise exponential distributions to approximate survival distributions, or the two-piece double exponential distribution or asymmetrical Laplace distribution (JKB, vol 2, p193).

2. Some Examples

Hybrid distributions arise whenever crossing a threshold brings consequences. One may think of income tax thresholds or of the differing rates of interest applicable to a positive or negative bank balance.

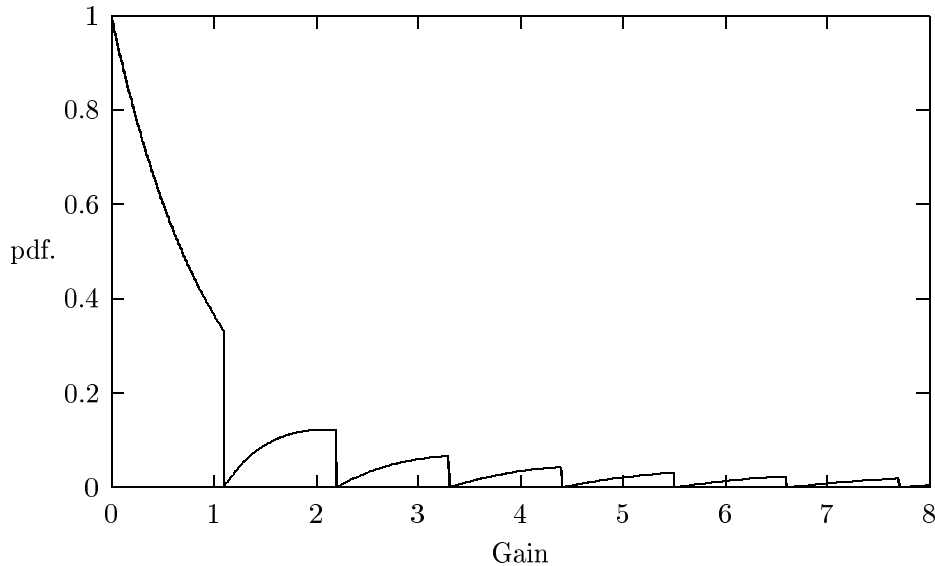
Consider a series of trials, each trial resulting in an exponentially distributed random variable, which could be the gain or loss from a gambling or entrepreneurial activity. We stop when the total gain has reached a threshold a , or after n trials, whichever occurs first. We could instead think of the time to arrival of the last successful purchaser, if there are n items for sale, the vendor stops trading after the departure of the first customer after time a , and purchasers arrive in a Poisson process. In reliability, a system might be used for a maximum of n cycles, or until wear exceeded a threshold.

Take the exponential random variable to have expectation of unity. This procedure results in a hybrid distribution for the resulting total gain X (or arrival time of the last successful purchaser), with

$$f(x) = \begin{cases} x^{n-1} \exp(-x)/(n-1)! & \text{if } 0 < x < a \\ c \exp(-x) & \text{if } x > a, \end{cases} \quad (1)$$

where $c = \sum_{k=0}^{n-1} a^k/k!$.

A distribution with cutpoints at all integer values arises from stopping gambling when the average gain per trial falls below $a > 1$. This distribution is shown in the figure for $a = 1.1$.



A discrete hybrid distribution was used by Baker (2000) to model the observed duration of an infection which was detected by periodic sampling with some probability $p < 1$ of successful detection. It is the distribution of the number of tails resulting from tossing a coin until either m heads turn up or until n tosses have been made. The number of tails is then a random variable from a hybrid binomial-negative binomial distribution.

REFERENCES

Baker, R.D., (2000). Application of a new discrete distribution, *Journal of Applied Statistics 2000*, **27**, 5-21.

Johnson, N, L., Kotz, S. and Balakrishnan, N. (1995). *Continuous univariate distributions, volume 2, 2nd Ed., John Wiley and Sons.*

RESUME

The concept of a hybrid distribution is introduced as a distribution whose pdf or pmf changes functional form at one or more cutpoints. Some examples are given. There are many possible applications, for example in reliability. As these distributions typically show an increasing hazard function, that suddenly changes at the cutpoint, they could be used to model survival before and after the onset of a life-threatening disease or medical condition. As distributions of gains from gambling or entrepreneurial activity, they would find applications in finance.