A Multivariate Growth Curve Model with Differing Numbers of Random Effects

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Suppose that we obtain repeated measurements of m response variables on each of p occasions (or treatments) for each of N individuals and that we can use observations of r covariates for each individual. Let $\boldsymbol{x}_j^{(g)}$ be an mp-vector of measurements on the j-th individual in the g-th group arranged as $\boldsymbol{x}_j^{(g)} = (x_{11j}^{(g)}, ..., x_{1mj}^{(g)}, ..., x_{p1j}^{(g)}, ..., x_{pmj}^{(g)})'$, and assume that $\boldsymbol{x}_j^{(g)}$'s are independently distributed as $N_{mp}(\mu_j^{(g)}, \Omega)$, where Ω is an unknown $mp \times mp$ positive definite matrix, $j = 1, \dots, N_g$, $g = 1, \dots, k$. Further, we assume that mean profiles of $\boldsymbol{x}_j^{(g)}$ are m-variate growth curves with r covariates, i.e., $\mu_j^{(g)} = (B' \otimes I_m)\xi^{(g)} + \Theta' \boldsymbol{c}_j^{(g)}$, where B is a $q \times p$ within-individual design matrix of rank q ($\leq p$), $B' \otimes I_m$ is the Kronecker product of B' and the $m \times m$ identity matrix, $\boldsymbol{c}_j^{(g)}$'s are r-vectors of observations of covariates, $\xi^{(g)}$'s are mq-vectors of unknown parameters, Θ is an unknown $r \times mp$ parameter matrix. Let $X = [\, \boldsymbol{x}_1^{(1)}, \dots, \boldsymbol{x}_{N_1}^{(1)}, \dots\dots, \boldsymbol{x}_1^{(k)}, \dots, \boldsymbol{x}_{N_k}^{(k)} \,]'$, $N = N_1 + \cdots + N_k$. Then the model of X can be written as

$$(1) \qquad X \sim N_{N \times mp}(A\, \Xi (B \otimes I_m) + C\Theta, \;\; \Omega \otimes I_N),$$

where A is an $N \times k$ between-individual design matrix, $C = [\, \boldsymbol{c}_1^{(1)}, \dots, \boldsymbol{c}_{N_1}^{(1)}, \dots\dots, \boldsymbol{c}_1^{(k)}, \dots, \boldsymbol{c}_{N_k}^{(k)} \,]'$ is a fixed $N \times r$ matrix of covariates, rank $[\, A, C \,] = k + r$ ($\leq N - p$), $\Xi = [\, \xi^{(1)}, \dots, \xi^{(k)} \,]'$ is an unknown $k \times mq$ parameter matrix. Without loss of generality, we may assume that $BB' = I_q$. The mean structure of (1) is a mixed MANOVA-GMANOVA model, and the GMANOVA portion is an extension of Potthoff and Roy (1964) to the multiple-response case. In this paper we consider a family of covariance structures

$$(2) \qquad \Omega_s = (B_s' \otimes I_m)\Delta_s (B_s \otimes I_m) + I_p \otimes \Sigma_s, \qquad 0 \leq s \leq q,$$

which is based on random-coefficients models with differing numbers of random effects (see Lange and Laird (1989)), where Δ_s and Σ_s are arbitrary $ms \times ms$ positive semi-definite and $m \times m$ positive definite matrices respectively, B_s is the matrix which is composed of the first s rows of B. This family is a generalization of a multivariate random-effects covariance structure proposed by Reinsel (1984). We derive a canonical form of the model (1). First we propose test statistics for the hypothesis

$$(3) \qquad H_{0s}: \; \Omega = \Omega_s \qquad \text{vs.} \qquad H_{1t}: \; \Omega = \Omega_t$$

in the model (1), where $1 \leq s < t \leq q$. Since the exact likelihood ratio (= LR) statistic for the hypothesis is complicated, it is suggested to use a modified LR statistic, which is the LR statistic for a modified hypothesis. An asymptotic expansion of the null distribution of the statistic is obtained. By making this strong assumption that $\Omega = \Omega_s$, we can expect to have efficient estimators. Next we obtain the maximum likelihood estimators (= MLE's) of unknown mean parameters under the covariance structures (2). In comparison with the MLE of Ξ when no special assumptions about Ω are made, we show how much gains can be obtained for the maximum likelihood estimation of Ξ by assuming that Ω has the structures (2).

REFERENCES

Lange, N. and Laird, N. M. (1989). The effect of covariance structure on variance estimation in balanced growth-curve models with random parameters. Journal of the American Statistical Association 84, 241–247.

Potthoff, R. F. and Roy, S. N. (1964). A generalized multivariate analysis of variance model useful especially for growth curve problems. Biometrika 51, 313–326.

Reinsel, G. (1984). Estimation and prediction in a multivariate random effects generalized linear model. Journal of the American Statistical Association 79, 406–414.

FRENCH RÉSUMÉ

Cet article traite des modèles de courbe de grandissement à plus de trois dimensions avec covariance et effets aléatoires. Ce modèle est un modèle mixte MANOVA-GMANOVA possédant une structure à covariance à effets aléatoires. Sont proposés ici des statistiques d'essai pour une hypothèse générale relative à la justesse d'une famille des structures à covariance. Ont ainsi été obtenues des statistiques de LR relatives à l'hypothèse émise et à son expansion asymptotique. Les MLE des paramètres moyens inconnus sont obtenus à partir des structures à covariance et l'efficacité du MLE est mise à discussion.