What Does the Standardized Mortality Ratio Compare?

Yoshisada Shibata
Department of Radiation Epidemiology
Nagasaki University School of Medicine
1-12-4 Sakamoto, Nagasaki 852-8523, Japan
yshibata@net.nagasaki-u.ac.jp

1. Introduction

The standardized mortality ratio (SMR)[1] is frequently used in public health society as a measure for comparing the mortality among several groups. Some authors recommend the use of SMR in comparison of the mortality among small areas even when the age-adjusted mortality rate is available. The present paper discusses the issues associated with the use of SMR for comparing groups.

2. Standardized mortality ratio

We introduce the following notations for discussion:

n_i: population of i-th age class in the target group;

d_i: number of annual deaths in the i-th age class of the target population;

p_i: annual mortality rate in the i-th age class of the target population; and

p_{i0}: standard annual mortality rate in the i-th age class.

The SMR is defined as:

$$SMR = \Sigma\, d_i / \Sigma\, n_i p_{i0}$$

(usually multiplied by 100) and the hypothesis test is conducted. However, the hypothesis to be tested is usually not stated explicitly. We first clarify the hypothesis to examine the properties of the hypothesis test based on the SMR.

It is natural to consider that the SMR estimates the population SMR defined by:

$$\xi = \Sigma\, n_i p_i / \Sigma\, n_i p_{i0}.$$

Then the usual hypothesis test based on the SMR is that about the null hypothesis that $\xi = 1$. Comparison of the two groups by SMR is a test of the null hypothesis that $\xi^{(1)} = \xi^{(2)}$. Since ξ is a ratio of two crude mortality rates, it usually varies with the age distribution of the group. Hence interpretation of the hypothesis that $\xi^{(1)} = \xi^{(2)}$ is not easy.

3. Hypothesis test

For simplicity, we consider a case to test the null hypothesis H_0: $\xi = 1$ against H_1: $\xi > 1$. Under the assumption that the number of deaths follows a Poisson distribution, the numerator of the SMR has the Poisson distribution with mean $\Sigma\, n_i p_i$ and the normal approximation leads us to rejecting H_0 if the following holds:

$$SMR > 1 + u_\alpha / \Sigma\, n_i p_{i0},$$

where u_α is the upper α-point of the standard normal distribution. The power of the test is

$$Power = 1 - \Phi\left[\sqrt{\sum_{i=1}^{K} n_i p_{i0}}\left(\frac{1}{\sqrt{\xi}} - \sqrt{\xi}\right) + \frac{u_\alpha}{\sqrt{\xi}}\right],$$

where Φ denotes the standard normal distribution function. The power thus depends on $\Sigma\, n_i p_{i0}$ as well as ξ.

4. Numerical example

To examine the effects of age distribution and standard mortality rate on the population SMR and the power of the test, we made 47 groups of size 10,000 on the basis of 1990 census population of 47 prefectures of Japan. We calculated the population SMR and the power of the test at ξ =1.2 using the age specific male and total mortality rates of 1990 vital statistics as mortality rate in 47 groups and the standard mortality rate, respectively. The results are shown in Fig. 1.

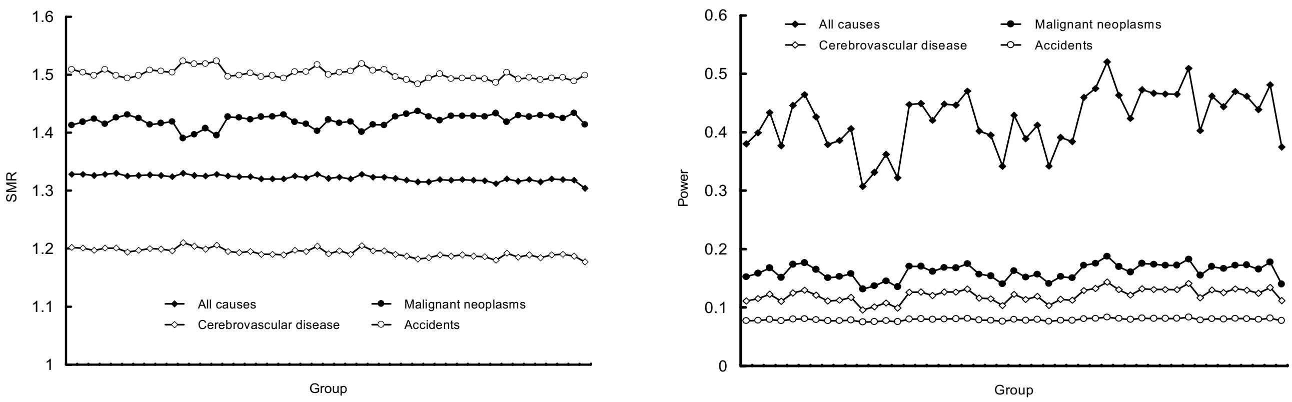

Fig 1. Effects of age distribution and standard mortality rate on the population SMR (left panel) and the power of the test at ξ =1.2 (right panel).

Since the mortality rates in 47 groups were set same, the difference in the population SMR shown in the left panel of Fig 1 exactly reflects the difference in the age distribution among 47 groups. The right panel of Fig. 1 indicates that the power of the test is prone to be effected much by the age distribution of the group and the standard mortality rate.

5. Conclusion

The SMR is not an adequate measure for comparing the mortality among two or more groups unless their age distributions do not differ much.

REFERENCE

1. Lilienfeld AM, Lilienfeld DE. *Foundations of Epidemiology.* Oxford University Press, 1980.