Selection Procedures to Select Populations Better than a Control

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Introduction and Summary

Let $\Pi_0, \Pi_1, .., \Pi_k$ be (k+1), $k \geq 2$ independent populations. The population Π_0 is assumed to be control population and populations $\Pi_1, .., \Pi_k$ are the treatment populations. In this paper, we define bestness in terms of location parameters and the problem of selecting all populations better than the control is considered in two cases. In case (i) assume that the population Π_i has the absolutely continuous distribution function $F_i(x)=F(x-\theta_i)$, where θ_i is the location parameter, i=0,1,..,k and F(.) is an (unknown) absolutely continuous distribution function. The treatment population Π_i is said to be better than the control population Π_0 if $\theta_i \geq \theta_0$, i=1,..,k. The goal is to select a subset of the k treatment populations which contains all the populations better than the control. Any such selection is called a correct selection (CS). In case (ii) the underlying assumption is that the (k+1) populations differ in their location parameters and have F(0)=p, so that θ_i, the location parameter of the ith population, is its pth quantile, i=0,1,..,k.

Rizvi, Sobel and Woodworth(1968) proposed non-parametric ranking procedures for comparison of treatment populations with a control in terms of α-quantiles. Deshpande and Mehta(1983) proposed procedures while comparing populations in terms of distribution functions. Gill and Mehta(1993) developed selection procedures for selecting populations better than a control population while restricting to only scale parameters.

Lehmann(1963), Bartlett and Govindarajulu(1968), Puri and Puri(1969) developed selection procedures based on joint ranking of sample observations from all the populations. However, Rizvi and Woodworth(1970) provide counter examples that the procedures based on joint ranking do not control the probability of correct selection over both the slippage parametric configuration (used under indifference-zone approach) as well as the entire parametric space (used under subset selection approach). Hsu(1981) used pairwise ranking to propose subset selection procedures and has shown that these procedures control the PCS over the entire parametric space.

In this paper we have used two-sample statistics for proposing subset selection procedures. These procedures control the PCS over the entire parametric space and satisfy the P^*-condition: $P_q[CS] \geq P^*$ for all $q \, \varepsilon \, \Omega$,
where $\Omega = \{q : q = (\theta_0, \theta_1, .., \theta_k), -\infty < \theta_i < \infty\}$ is the parametric space and $2^{-k} < P^* < 1$.

The proposed procedures are shown to be strongly monotone. Approximate implementation of the procedures, with the help of existing tables, is discussed. Simulation study is carried out to see the relative performance of these procedures. As a measure of "goodness" of a subset selection procedure, we use the ratio of the estimated expected subset size E(S) to the estimated probability of correct selection $P^{\wedge}(CS)$, i.e. E(S)/ $P^{\wedge}(CS)$. A rule is said to be "better" than a rule R^* if the ratio for R is less than the ratio of R^*. The relative efficiency of the procedure R_1 relative to R_2 is an inverse ratio of the measures of goodness, i.e.,

$$e(R_1,R_2) = E(S/R_2)\, P^{\wedge}(CS/R_1)/\, E(S/R_1)\, P^{\wedge}(CS/R_2).$$

The values of $e(R_1,R_2)$ for different parametric configurations of families of distributions under various chioces of n(n=6(2)40) are computed, with interesting results.

REFERENCES

Bartlett,N.S. and Govidarajulu,Z.(1968). Some distribution-free statistics and their application to the selection problem. *Annals of Institute of Statistical Mathematics 20, 79-97.*

Deshpande,J.V. and Mehta,G.P.(1983). Non-parametric procedures to select populations better than a standard. *Sankhya, Series B 45, 330-334.*

Gill,A.N. and Mehta,G.P.(1993). Selecting populations better than a control: The scale parameter case. *Statistics and Decisions 11, 251-271.*

Hsu,J,C.(1981). A class of non-parametric subset selection procedures. *Sankhya, Series B 43, 235-244.*

Lehmann,E.L.(1963). A class of selection procedures based on ranks. *Math. Ann. 150, 268-275.*

Puri,P.S. and Puri,M.L.(1969). Multiple decision procedures based on ranks for certain problems in analysis of variance. *Annals of Mathematical Statistics 40, 619-632.*

Rizvi,M.H., Sobel,M. and Woodworth,G.G.(1968). Non-parametric ranking procedures for comparison with a control. *Annals of Mathematical Statistics 39, 2075-2093*.

Rizvi,M.H., and Woodworth,G.G.(1970). On selection procedures based on ranks: Counter examples concerning least favorable configurations. *Annals of Mathematical Statistics 41, 1942-1951.*