Bayesian Special Type Double Sampling Plan with Two Decision Criteria for Discrete Prior Distribution

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ABSTRACT

This paper presents Bayesian Sampling Plan based on Special Type of Double Sampling (STDS) Plan. The selection of parameters for STDS plan through Bayes solution considering double binomial as prior distribution is presented. Illustration is provided for easy selection of plan parameters.

INTRODUCTION:

To evaluate the minimum expected value for desirable lesser cost function, it is appropriate to consider some prior distribution for the quality characteristic or lot quality.

Calvin (1984) provides procedures and tables for implementing Bayesian Sampling Plans. A set of tables presented by Oliver and Springer (1972) are based on the assumption of a Beta-prior distribution with specific posterior risk to achieve minimum sample size, which avoids the problem of estimating cost parameters. Hald (1981) has provided an excellent comparison on classical and Bayesian theory and methodology for attributes Sampling Plans.

When Sampling plans are set for product characteristics that involves costly or destructive testing by attributes, it is the usual practice to use Single Sampling plan with acceptance number c=0 and c=1. But the OC curves for Single Sampling Plan with c=0 and c=1 leads to conflicting interest between the producer and consumer. Here c=0 favours consumer and c=1 favours producer. Govindaraju (1984) has proposed the Special Type of Double Sampling (STDS) Plan to avoid such shortcomings

Model under consideration:

Let N and n denote the lot size and sample size X and x denote the number of defectives in the lot and sample respectively.

The linear cost function is

$I(N,n,X,x) = nS_1 + xS_2 + (N-n)\, A_1 + (X-x)\, A_2,$ for acceptance

$= nS_2 + xS_2 + (N-n)\, R_2 + (X-x)\, R_2,$ for rejection ..(1)

where nS_1 denotes the cost of inspection and xS_2 denotes the cost proportional to the number of defective item in the sample. In fact S_1 includes sampling and testing cost per item and S_2 denotes additional cost for an inspected defective item including the repair cost per item if in case the defective items found in the sample which are repaired. Thus the cost $n_1S_1 + n_2S_2$ associated with the sample provide costs for sampling inspection.

Cost of acceptance is given as (N-n) A_1 + (X-x) A_2 and Cost for outright rejection is (N-n) R_1 +(X-x) R_2.The parts (N-n) A_1 and (N-n)R_1 are proportional to the number of items in the reminder of the lot and A_1 and R_2 are usually zero, generally negligible. The parts (X-x) A_2 and (X-x)R_2 are proportional to the number of defective items accepted and hence A_2 and R_1 are often considerable.

From Hald (1981) the cost function stated in (1) becomes

$I = n(S_1 + S_2p) + (N-n)(A_1 + A_2p)$, for acceptance ....(2)

$I = n(S_1 + S_2p) + (N-n)(R_1+R_2p)$, for rejection ....(3)

The average cost can be written as

$K(N,n,n_2\ p)= n(S_1 + S_2\ p) + (N-n)(A_1+A_2p)\ P_a(p)+[(R_1+R_2p)(1-P_a(p)]$ ..(4)

where $P_a(p) = (1-p)^n + n_2p\,(1-p)^{n-1}$ ....(5)

The double binomial distribution is a weighted average of two binomials with parameters p' and p'' ,such that p' < p'' and weights w_1 and w_2 such that $w_1+w_2 =1$.

Double binomial bayes solution

A necessary condition for a sampling plan to exist under the double binomial distribution is that $p' < p_r < p''$ where p_r is a break-even quality. Assuming that $p' < p_r < p''$, the standardized cost function can be written as

$R(N,n_1,n_2) = n+(N-n)[V_1(1-P_a(p'))+V_2\ P_a(p'')]$ ....(6)

Where $V_1 = w_1\ (p_r - p')$ and $V_2 = w_2\ (p_r - p'')$ ....(7)

It is required to obtain (n_1,n_2) which minimizes equation (6). The necessary condition for such sampling plan to exit is that $V_1>0$, $V_2>0$, $p' <p_r<p''$.

According to Hald (1981) the value for (n_1,n_2) minimizing R must satisfy the following conditions

$\nabla_{n_2} R(N,n,n_2 -1) \le 0 \le \nabla_{n_2} R(N,n,n_2)$, $0<n_2 < n$ .....(8)

$\nabla_n R(N,n-1,n_2) \le 0 \le \nabla_n R(N,n,n_2)$, $0<n < N$ .....(9)

By substituting equation (5) in equation (6) and solving the inequality mentioned in equation (9) one can get

n=-1/2+(ln(V2/V1)+ln(p''/p')+ln((1-p''+n2p'')/(1-p'+n2p'))+ln((1-p')/(1-p'')))/ln((1-p')/(1-p''))

Similarly one can solve equation (8) for obtaining parameters. By giving various values for n_2 , one can compute the values for n with specified V_1,V_2,p' and p'' .

Example: For specified p'=0.001, p''=0.1,V_1=0.00891and V_2= 0.0009,and n_2 = 15, the value of n can be obtained as n=31 and n_1 =16.Thus the parameters of the STDS plan are n_1 = 16 and n_2 = 15.

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RESUME

Ms. Deepa Menon is a research schilar mainly working in the field of acceptance sampling plans and has presented papers in many international and national conferences in the field of product control.