State Space Representation for Non Linear Spatio-Temporal Models

Mauro Coli – Eugenia Nissi
Dipartimento di Metodi Quantitativi e Teoria Economica
Facoltà di Economia
Viale Pindaro, 42 –65127 Pescara Italy
coli@dmqte.unich.it – nissi@dmqte.unich.it

1. Introduction

In this paper we propose a state-space representation for spatio-temporal bilinear models which can be used to model space time series which exhibit non linear behaviour (e.g. environmental data).

The Spatio-Temporal Bilinear model is shown to be an extension of space-time autoregressive moving average models (STARMA) and a special form of multiple bilinear model.

In particular we focus on the markovian representation of the above mentioned class of models and on the possible application of the Extended Kalman Filter.

2. Vectorial Space-Time Bilinear Models

Linear space-time models based on the ordering and weighting schemes are known as spatio-temporal autoregressive moving average models (STARMA) have been developed since the 1975 (Cliff and Ord 1975).

The *STARMA (p,* λ*;q,m)* models can be represented as follows:

$$\mathbf{y}(t)=\sum_{k=0}^{p}\sum_{l=0}^{\lambda(k)}\phi_{kl}\mathbf{W}^{(l)}\mathbf{y}(t-k)+\sum_{k=0}^{q}\sum_{l=0}^{\mathbf{m}(k)}\theta_{kl}\mathbf{W}^{(l)}\varepsilon(\mathrm{t+k})+\varepsilon(t)$$

where:

- $\mathbf{y}(t)$ is the (R×1) vector of the variables (with zero mean) observed in each of the R zones in which the territory under study is subdivided;
- p and q are the temporal orders, respectively, of the auto-regressive and moving-average components;
- λ_k and m_j are the spatial orders, respectively, of the k-th auto-regressive and of the i-th moving-average components;
- ϕ_{kl} is the auto-regressive parameter at temporal lag *k* and spatial lag *l*;
- θ_{ij} is the moving-average parameter at temporal lag *i* and spatial lag *j*;
- $\mathbf{W}^{(l)}$ is the (R×R) weights matrix (normalised by row), at spatial lag (l): the weights w_{ij} are equal to 1 if the zones i and j are neighbours at spatial lag (.), and to 0 otherwise;
- $\varepsilon(t)$ is the (R×1) vector of the error terms at time t; it is assumed to be normally distributed.

Dai and L. Billard (1998) has shown that the above models can be regarded as a special case of Bilinear Space-Time Models:

$$\mathbf{y}(t)=\sum_{k=0}^{p}\sum_{l=0}^{\lambda(k)}\phi_{kl}\mathbf{W}^{(l)}\mathbf{y}(t-k)+\sum_{k=0}^{q}\sum_{l=0}^{\mathbf{m}(k)}\theta_{kl}\mathbf{W}^{(l)}\varepsilon(\mathrm{t+k})+\sum_{i=1}^{r}\sum_{j=1}^{s}\sum_{m=0}^{\lambda_i}\sum_{n=0}^{m_i}\beta_{mn}^{ij}\left\{\mathbf{W}^{(l)}\mathbf{y}(t-k)\right\}\times\left\{\mathbf{W}^{(l)}\varepsilon(t-k)\right\}+\varepsilon(t)$$

where
r is the autoregressive order in the bilinear part;
s is the moving-average term in bilinear part;
x_i is the spatial order of the autoregressive term in the bilinear part at the temporal lag i;
m_j is the the spatial order of the moving average term in the bilinear part at temporal lag j;
b_{mn}^{ij} is the bilinear parameter at the temporal lags i and j for the autoregressive and moving average term respectively.

Obiously if b_{mn}^{ij} is equal to 0 the STARMA models are a simple case of Bilinear Space-Time models.

3. State Space Representation for Bilinear Models.

It seems opportune to underline that the structure of the Bilinear Space-Time models is formally concurrent with that of BARMA models.
From the above considerations, it becomes natural to extend the state space methodology for BARMA models to this models.
In particular, model can be expressed in a more compact notation through the following expressions

$$\mathbf{X}(t)=F\ \mathbf{X}(t-1)+\mathbf{C}\ \mathbf{V}(t)$$

$$\mathbf{Y}(t)=\mathbf{H}\ \mathbf{X}(t-1)+\mathbf{W}(t)$$

where X_t is the state vector representing the state of the process at time t, F is the transition matrix defining how the process progresses from one point to another, **C** and **H** are two matrices and **W**(t) and **V**(t) are sequences of normally distributed independent random variables with zero mean and covariance equal to $\mathbf{Q}_w$ and $\mathbf{Q}_v$.
In this settings, the estimation of the state vector can be obtained using the Kalman Filter, in particular due to the not linear nature of the system we can use the Extended Kalman Filter.

REFERENCE

Cliff A. D., Haggett P., Ord J.K., Bassett K.A., Davies R. B: (1975), *Elements of Statistical Structure : a Quantitative Approach*, Cambridge University Press.

Dai Y., Billard L. (1998), A Space-time Bilinear Models and its Identification, *Journal of Time Series Analysis*, Vol. 19, n.6, 657-679.

Kalman R.E. (1960), A New Approach to Linear Filtering and Prediction Problem, Transaction of the ASME- Journal of Basic Engineering, 35-45.

RESUME

Dans cet article nous proponons un modèle pour l' analyse des séries spatio-temporelle. Le modèle est a particulier cas de STARMA modèle.