Better Nonparametric Bootstrap Confidence Limits for Some Process Capability Indices

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Abstract

The index C_{pmk} is the third generation process capability index. This index is more powerful than two useful indices C_p and C_{pk}. If a process is clearly nonnormal, there is some questions as to whether any process index is valid or should even be calculated. However, bootstrap method could be studied for statistical inference.

In this paper, we study bootstrap inference for our process capability indices C_p, C_{pk} and C_{pmk}. Having studied bootstrapping our process capability index C_{pmk}, we construct six bootstrap confidence intervals and compare their performances for our process index. That is, we examine better nonparametric bootstrap confidence intervals for our process capability index C_{pmk} under some nonnormal distributions.

1. Introduction

Process capability indices, whose purpose is to provide a numerical measure on whether a production process is capable of producing items satisfying the quality requirements preset by the designer, have received substantial attention in the quality control and statistical literature. The two most widely used capability indices are $C_p = \frac{USL-LSL}{6\sigma}$, and $C_{pk} = \min\left\{\frac{USL-\mu}{3\sigma}, \frac{\mu-LSL}{3\sigma}\right\}$,

where USL is the upper specification limit, LSL is the lower specification limit, μ is the process mean, and σ is the process statndard deviation. While the index C_p reflects only the magnitude of the process variation, the index C_{pk} takes into account the process variation as well as the location of the process mean relative to the specification limits. Also, to obtain more sensitive capability index than C_{pk} and C_{pm}, Pearn et al.(1992) introduced the third process capability index C_{pmk} as follows :

$$C_{pmk} = \frac{\min(USL - \mu, \mu - LSL)}{3\sqrt{E(X-T)^2}}$$

In general, the calculation of various lower confidence limits assume a normally distributed process, and, as Gunter(1989) has noted, many real world processes are not normally distributed and this departure from normality may be hard to detect. This could potentially affect both the estimates of the indices and the lower confidence limits based on these estimates. Efron(1979) introduced and developed the nonparametric, but computer intensive, estimation method called bootstrap. In particular, Efron and Tibshirani(1986) further develop three types of bootstrap confidence intervals: the SB confidence interval, the PB confidence interval, and the BCPB confidence interval. Franklin and Wasserman(1991) presented an inital study of the properties of these three bootstrap confidence intervals for C_{pk}. Also, Franklin and Wasserman(1992) have studied bootstrap lower confidence limits for capability indices. In particular, there is no result on statistical inference for the index C_{pmk} because of computing complexity. With this background, we focus on bootstrapping some process capability indices.

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