Bootstrapping Vector-valued Process Capability Indices

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1. Introduction

Process capability index(PCI) is used to determine whether a process is capable of producing items within a specified tolerance. In general, the quality of a product can be judged on the basis of several characteristics, each of which must satisfy certain specifications. The quality of the product thus depends on their combined effect rather than their individual performances. Since these characteristics are measured on the same individual, it is realistic to consider them to be related and that they are jointly distributed. It is a risky undertaking to represent variation of even a univariate characteristic by a single index. The possibility of hiding important information is much greater when multivariate characteristics are under consideration, and the desirability of using vector-valued process capability indices arises quite naturally. Also, multivariate quality control is particularly important in the present day context as automatic inspection procedures have made it easy to measure many characteristics on each manufactured item. Alt and Smith(1988) have given an excellent review of the multivariate process control techniques currently available. The important problem of assessing the process capability based on several characteristics has received scant attention until recently. The work of Kocherlakota,S. and Kocherlakota,K.(1991) provides the joint distribution of $\hat{C}_{px}$ and $\hat{C}_{py}$ under bivariate normal distribution $BN(\mu_1, \mu_2, \sigma_1^2, \sigma_2^2, \rho)$. Vector-valued process capability indices might be more useful than multivariate process capability indices. But not much research for robust estimation is in process for vector-valued process capability indices. Therefore, bootsrap method could be studied for statistical inference(Efron,B.(1979) and Beran,R.J.(1984)).

In this paper we study vector-valued process capability indices with bootstrap method.

First, we are interested in the statistical estimation of two vector-valued PCIs $\mathbf{C_p} = (C_{px}, C_{py})$ and $\mathbf{C_{pm}} = (C_{pmx}, C_{pmy})$ considering basic process capability indices C_p and C_{pm} . Some asymptotic distributions of two plug-in estimators $\hat{\mathbf{C}}_\mathbf{p} = (\hat{C}_{px}, \hat{C}_{py})$ and $\hat{\mathbf{C}}_\mathbf{pm} = (\hat{C}_{pmx}, \hat{C}_{pmy})$ are derived. Obtaining the asymptotic distribution of two bootstrap estimators $\hat{\mathbf{C}}^*_\mathbf{p} = (\hat{C}^*_{px}, \hat{C}^*_{py})$ and $\hat{\mathbf{C}}^*_\mathbf{pm} = (\hat{C}^*_{pmx}, \hat{C}^*_{pmy})$ with our bootstrap algorithm, we will provide the consistency of our bootstrap for our statistical inference.

Second, for our vector-valued PCIs $\mathbf{C_p} = (C_{px}, C_{py})$ and $\mathbf{C_{pm}} = (C_{pmx}, C_{pmy})$, we construct asymptotic confidence region and some bootstrap confidence regions based on various limiting distributions of plug-in estimators and bootstrap estimators. Of course, these asymptotic confidence regions can be easily obtained more conveniently without any distribution assumption. Through monte-carlo simulation , we examine their small sample properties and compare their performances under bivariate normal distribution and some bivariate nonnormal distributions, constructing better confidence regions.

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