Statistical Analyses of Round Robin Interaction Data

Tim B. Swartz
Simon Fraser University, Department of Statistics and Actuarial Science
8888 University Drive
Burnaby, Canada
tim@stat.sfu.ca

Paramjit S. Gill
Okanagan University College, Department of Mathematics and Statistics
3333 College Way
Kelowna, Canada
pgill@okanagan.bc.ca

1. Introduction

Round robin interaction data arise when individuals from a group of subjects interact with one-another. An interaction between two subjects produces a pair of outcomes, one for each subject. There are many types of scientific investigations that give rise to round robin interaction data, and as such, there is a need for good methods of statistical analysis.

Suppose that a round robin design involves m subjects where subjects i and j meet n_{ij} times, $i \neq j$. When subjects i and j meet on the kth occasion, we obtain a pair of observations y_{ijk} and y_{jik} as a realization of continuous random variables. Here y_{ijk} represents the response of subject i as an *actor* towards subject j as a *partner* on the kth occasion; and in y_{jik}, the roles are reversed. Warner *et al.* (1979) proposed the general round robin model

$$\begin{aligned} y_{ijk} &= \mu + \alpha_i + \beta_j + \gamma_{ij} + \varepsilon_{ijk} \\ y_{jik} &= \mu + \alpha_j + \beta_i + \gamma_{ji} + \varepsilon_{jik} \\ k &= 1, ..., n_{ij};\ 1 \leq i \neq j \leq m. \end{aligned} \tag{1}$$

In this model μ is the general mean; α_i represents the effect of subject i as an actor; β_j is a partner effect due to subject j; γ_{ij} is an interaction effect representing the special adjustment which subject i makes for subject j; ε_{ijk} represents the error term which picks up the measurement error and/or variability in behaviour at different occasions. Note that if $n_{ij} < 2$ for some i, j, the data does not distinguish between γ_{ij} and ε_{ijk}. Except for the general mean μ, all parameters in model (1) are assumed to be random variables and are known as the random effects. Based on assumptions on the mean and covariance structure of the parameters, various researchers have considered model (1) and related models using ANOVA and maximum likelihood methods.

2. Bayesian Analysis

Gill and Swartz (2001) consider a fully Bayesian analysis of model (1) which yields a number of advantages. Letting $\mu_{ij} = \mu + \alpha_i + \beta_j + \gamma_{ij}$, we assume conditionally

$$\begin{pmatrix} \alpha_i \\ \beta_i \end{pmatrix} \sim \text{Normal}_2 \left[\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \Sigma_1 \right],$$

$$\begin{pmatrix} \gamma_{ij} \\ \gamma_{ji} \end{pmatrix} \sim \text{Normal}_2 \left[\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \Sigma_2 \right],$$

$$\begin{pmatrix} y_{ijk} \\ y_{jik} \end{pmatrix} \sim \text{Normal}_2 \left[\begin{pmatrix} \mu_{ij} \\ \mu_{ji} \end{pmatrix}, \Sigma_3 \right]$$

where $k = 1, \ldots, n_{ij}$, $1 \le i \ne j \le m$ and

$$\Sigma_1 = \begin{pmatrix} \sigma_\alpha^2 & \sigma_{\alpha\beta} \\ \sigma_{\alpha\beta} & \sigma_\beta^2 \end{pmatrix}, \quad \Sigma_2 = \begin{pmatrix} \sigma_\gamma^2 & \sigma_{\gamma\gamma} \\ \sigma_{\gamma\gamma} & \sigma_\gamma^2 \end{pmatrix}, \quad \Sigma_3 = \begin{pmatrix} \sigma_\varepsilon^2 & \sigma_{\varepsilon\varepsilon} \\ \sigma_{\varepsilon\varepsilon} & \sigma_\varepsilon^2 \end{pmatrix}.$$

In addition, we assume

$$\begin{aligned} \mu &\sim \text{Normal}[\tau, \sigma_\mu^2], \quad \Sigma_1^{-1} \sim \text{Wishart}_2[(\rho_0 r_0 I)^{-1}, \rho_0], \\ \tau &\sim \text{Normal}[\tau_0, \sigma_\tau^2], \quad \sigma_\mu^2 \sim \text{Inverse Gamma}[a_0, b_0] \end{aligned}$$

where the hyperparameters are set to give diffuse prior distributions for the parameters τ, σ_μ^2 and Σ_1. For Σ_2, we assume that $\sigma_\gamma^2 \sim \text{Exponential}[r_0]$ and $\sigma_{\gamma\gamma} \mid \sigma_\gamma \sim \text{Uniform}[-\sigma_\gamma^2, \sigma_\gamma^2]$. A similar prior structure is also imposed on Σ_3. These atypical bivariate priors are appealing as they are simply characterized by the single specified hyperparameter r_0. Moreover, $\text{E}(\sigma_\alpha^2) = \text{E}(\sigma_\beta^2) \approx \text{E}(\sigma_\varepsilon^2) = \text{E}(\sigma_\gamma^2) = r_0$ which suggests a commonality of magnitude amongst the various effects and we note that r_0 can often be chosen by the experimental structure.

After determining the full conditional distributions, the Gibbs sampling algorithm allows investigation of the posterior distribution.

REFERENCES

Warner, R.M., Kenny, D.A. and Stoto, M. (1979). A new round robin analysis of variance for social interaction data. *Journal of Personality and Social Psychology*, **37**, 1742-1757.

Gill, P.S. and Swartz, T.B. (2001). Statistical analyses for round robin interaction data. *Canadian Journal of Statistics*, **29**, To appear.

RESUME

Cet article concerne l'analyse de donneés issues de tournois à la ronde dans lesquels l'interaction des membres d'un groupe deux à la fois donne lieu à des paires de résultats, un pour chaque individu.