On choosing a parameter of image restoration filters

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1. Introduction

In this paper, we study a method of choosing parameters in image restoration filter. For an $N \times N$ image degraded by a linear blur and independent identically distributed zero mean additive noise, the imaging equation is

$$g = Af + \varepsilon, \tag{1}$$

where the vectors g, f, and n are $M^2 \times 1$ lexicographic orders of a degraded observed image, the $N^2 \times 1$ original image, and the noise, respectively. The $M^2 \times N^2$ degradation matrix A is ill conditioned.

It is shown that regularization is an effective method for obtaining solutions to problems. According the regularization approach, the solution of (1) is the minimization of

$$J_\lambda(f) = ||g - Af|| + \lambda f'Cf,$$

which yields the estimate

$$\hat{f} = (A'A + \lambda C)^{-1}g = B(\lambda)g,$$

where C is the regularization operator. The parameter λ controls the trade-off between accuracy of a restored image and the energy of noise passed through restoration. It is pointed out that

quality of restoration depends on the choice of parameters (Ogawa and Oja (1986)). Therefore, a lot of method for choosing parameters are proposed (Thompson et al.(1991), Galatsanos and Katsaggelos (1992), Archer and Titterington (1995)).

These methods are based on residual sum of squares between the original image and a degraded observed image. In this study, we propose a method based on the Kullback-Leibler information instead of the residual sum of squares.

2. Penalised likelihood estimation and image restoration

We assume that an additive noise ε in the Eq. (1) is normally distributed, that is,

$$\varepsilon \sim N(0, \sigma^2 I_{M^2}),$$

where σ^2 is unknown noise variance. Thus, the penalised likelihood function is

$$-\frac{M^2}{2}\log 2\pi - \frac{M^2}{2}\log\sigma^2 - \frac{1}{2\sigma^2}(g-Af)'(g-Af) - \frac{\gamma}{2}f'Cf,$$

where C is a prescribed nonnegative-definite matrix. Thus, the maximum penalised likelihood estimator is the solution of

$$\begin{aligned} \hat{f} &= (A'A + \hat{\sigma}^2\gamma C)g, \\ \hat{\sigma}^2 &= \frac{1}{M^2}(g-A\hat{f})'(g-A\hat{f}) \end{aligned}$$

REFERENCES

G. Archer and D. M. Titterington (1995). On some Bayesian / regularization methods for image restoration. *IEEE Trans. on Image Processing*, **4**(7), 989–995.

N. P. Galatsanos, A. K. Katsaggelos (1992). Methods for choosing the regularization parameter and estimating the noise variance in image restoration and their relation. *IEEE Trans. Image Processing*, **1**(3), 322–336.

E. Oja and H. Ogawa (1986). Parametric Projection Filter for image and signal restoration. *IEEE Trans. on Acoust., Speech & Signal Processing*, **ASSP-34**(6), 1643–1653.

A. M. Thompson, J. C.Brown, J. W. Kay, and D. M. Titterington (1991). A study of methods of choosing the smoothing parameter in image restoration by regularization. *IEEE Trans. on Pattern Analysis and Machine Intelligence*, **13**(4), 326–339.

RESUME

Dans ce papier, nous étudions une méthode de choisir des paramètres dans filtre de la restauration de l'image. Nous proposons une méthode basée sur le Kullback - Leibler renseignements au lieu de la somme du resudual de carrés.