Application of Closed Testing Procedure to a Bootstrap Multiple Comparisons

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1. The problem

The problem of multiple comparisons is familiar to most statisticians. One solution to that problem has been suggested in this paper.

We consider L samples of size $n_1, n_2, ..., n_L$ from L distributions, with expected values $m_1, m_2, ..., m_L$ are to be compared. The objective is to tell which $m_i^{'}$s are different and eventually rank them in descending order. The null and alternative hypotheses could be stated as

$$H_{Oij} : m_i \leq m_j ; \quad i.j = 1, 2, ..., L \text{ and } i \neq j$$
$$H_{Aij} : m_i > m_j ; \quad i.j = 1, 2, ..., L \text{ and } i \neq j$$

Let $H = \{ H_{01}, ..., H_{0n} \}$ be a set of all null hypotheses. Assume that H is closed under intersection, that is

$$\bigwedge_{i \neq j} H_{0i} \cap H_{0j} \in H$$

Now, any $H_{oj} \in H$ is rejected if and only if all hypothesis, H_{oi}, that are included in H_{0j} and belonging to H have been testing and rejected. The multip le significance level of this test is not greater than a.

2. The closed bootstrap multiple test procedures

The basic idea of the bootstrap multiple test procedure is to form all possible pairwise differences among the L sample means $\bar{y}_1, ..., \bar{y}_L$, and with a number of bootstrap samples determine whether the observed differences are likely to occur just by chance or if they imply differences between the corresponding true means.

Let $d_{ij} = m_i - m_j ; i, j = 1, 2, ..., L, \ i \neq j$ be the true differences and $d_{ij} = \bar{y}_i - \bar{y}_j$ be the sample differences.

Hypotheses could be stated as:

$$H_{0k}: d_k \leq 0; \; k = 1,..., K$$
$$H_{Ak}: d_k > 0; \; k = 1,..., K$$

where $K = L(L-1)$.

The hypotheses are now to be tested in the sequentially rejective manner.

Let y_k be the number of times the bootstrap differences. Whole procedure is repeated B times, where B is a rather large number e. g. 1000 to 10000.

The rule is, starting with $k = 1$:

$$\text{if } y_k / B \leq a \text{ reject } H_{0k} \text{ and test } H_{0k+1}$$
$$\text{if } y_k / B > a \text{ accept } H_{0i}, \; i > k.$$

Let $\Delta = \{d_{ij}\}$ be a set of K true differences. Δ is possible divide to Δ^+ including the positive elements and Δ^- including the negative and zero ones.

In terms of d_{ij} the null hypothesis is: $H_0 : d_{ij} \in \Delta^-$; for all $i \neq j$.

The set H of null hypotheses is then the possible decisions of Δ^- and Δ^+.

This set is obviously closed since the intersection between any two divisions results in a third one also included in *H*. Hence it is likely to keep the multiple level of significance at the predetermined value.

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RESUME

Le procédé multiple présenté dans cet article a un caractère de bootstrap, en remplaçant les dispositions théoriques oar leurs équivalents empiriques. On a démontré que c'est le procédé fermé et qu'il maintient un niveau multiple d'effectivité au niveau avant fixé.