Using BDS statistics to detect nonlinearity in time series

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1. Introduction

Chaos theory has captured attentions of many financial analysts and economists in these years. Chaos theory is based on the assumptions that the underlying system is a non-linear process, and the underlying system is a deterministic system. A number of recent studies have found strong evidence of nonlinearity in the short-term movements of asset returns (Hsieh, 1993).

BDS test was first devised by W.A. Brock, W. Dechert and J. Scheinkman in 1987 (Brock, Dechert & Scheinkman, 1987). BDS test is a powerful tool for detecting serial dependence in time series. It tests the null hypothesis of independent and identically distributed (I.I.D.) against an unspecified alternative. BDS test cannot test chaos directly, but only nonlinearity, provided that any linear dependence has been removed from the data (e.g. using traditional ARIMA-type models or taking a first difference of natural logarithms). Nevertheless, nonlinearity is one of the indications of chaos, we may use BDS test to detect such indication.

2. Mathematical details

BDS test employs the concept of spatial correlation from chaos theory. The computations of BDS test follow the following procedures:

1. Given a time series with N observations, which should be the first difference of the natural logarithms of raw data in time series.
$$\{x_i\} = [x_1, x_2, x_3, \ldots, x_N]$$
2. Select a value of m (embedding dimension), embed the time series into m-dimensional vectors, by taking each m successive points in the series. This converts the series of scalars into a series of vectors with overlapping entries.
$$x_1^m = (x_1, x_2, \ldots, x_m)$$
$$x_2^m = (x_2, x_3, \ldots, x_{m+1})$$
$$\vdots$$
$$x_{N-m}^m = (x_{N-m}, x_{N-m+1}, \ldots, x_N)$$
3. Compute the correlation integral, which measures the spatial correlation among the points, by adding the number of pairs of points (i, j), where $1 \le i \le N$ and $1 \le j \le N$, in the m-dimensional space which are "close" in the sense that the points are within a radius or tolerance ε of each other.
$$C_{\varepsilon,m} = \frac{1}{N_m(N_m - 1)} \sum_{i \neq j} I_{i,j;\varepsilon} \tag{1}$$
$$\text{where, } I_{i,j;\varepsilon} = 1 \quad \text{if } \left\| x_i^m - x_j^m \right\| \le \varepsilon$$
$$= 0 \quad \text{otherwise}$$
4. Brock, Dechert and Scheinkman (1987) showed that if the time series is I.I.D.
$$C_{\varepsilon,m} \approx [C_{\varepsilon,1}]^m \tag{2}$$
If the ratio $\frac{N}{m}$ is greater than 200, the values of $\frac{\varepsilon}{s}$ range from 0.5 to 2 (Lin, 1997) and the values of m are between two and five (Brock *et al.*, 1988), the quantity $[C_{\varepsilon,m} - (C_{\varepsilon,1})^m]$ has an asymptotic normal distribution with zero mean and a variance $V_{\varepsilon,m}$ defined as:

$$V_{e,m} = 4[K^m + 2\sum_{j=1}^{m-1} K^{m-j} C_e^{\ 2j} + (m-1)^2 C_e^{\ 2m} - m^2 K C_e^{\ 2m-2}] \tag{3}$$

Where, $K = K_e = \dfrac{6}{N_m(N_m - 1)(N_m - 2)} \sum_{i<j<N} h_{i,j,N;e}$; $h_{i,j,N;e} = \dfrac{[I_{i,j;e} I_{j,N;e} + I_{i,N;e} I_{N,j;e} + I_{j,i;e} I_{i,N;e}]}{3}$

5. The BDS test statistic can be stated as:

$$BDS_{e,m} = \frac{\sqrt{N}[C_{e,m} - (C_{e,1})^m]}{\sqrt{V_{e,m}}} \tag{4}$$

BDS test is a two-tailed test, we should reject the null hypothesis if the BDS test statistic is greater than or less than the critical values (e.g. if α=0.05, the critical value = ±1.96).

3. Data

The data set used in this study consists of daily Composite Indices of Shanghai Stock Exchanges (SHSE) and Shenzhen Stock Exchanges (SZSE). The data covers a 8-year period from 5 October 1992 to 29 December 2000, consisting of 2348 observations.

4. Empirical results

A level of significance (α) of 5% is taken in this hypothesis testing. Table 1 presents the test statistic in the BDS test for the returns series. The hypothesis of the test is as follow:

H_0: The data are independently and identically distributed (I.I.D.)

H_1: The data are not I.I.D.; this implies that the time series is non-linearly dependent if first differences of the natural logarithm have been taken

Table 1. BDS test results (logarithmic daily returns)

e/s	Embedding dimension (*m*)	BDS Test Statistics		e/s	Embedding dimension (*m*)	BDS Test Statistics	
		Shanghai	**Shenzhen**			**Shanghai**	**Shenzhen**
2	2	11.82	13.88	1	2	15.42	17.88
2	3	15.55	16.61	1	3	19.54	22.54
2	4	17.31	17.82	1	4	22.58	26.67
2	5	18.27	18.72	1	5	24.93	30.83
1.5	2	13.93	15.96	0.5	2	16.22	18.93
1.5	3	17.50	18.96	0.5	3	21.27	26.20
1.5	4	19.77	20.99	0.5	4	27.28	35.79
1.5	5	20.98	22.87	0.5	5	34.06	48.15

Table 1 indicates all the test statistics are greater than the critical values significantly. Thus, we should reject the null hypothesis of I.D.D. The results strongly suggest that the time series in both Chinese stock markets are non-linearly dependent, which is one of the indications of chaotic behavior.

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